DOCUMENT RESUME

ED 038 292 24 SE 008 176

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TITLE	The Identification and Encouragement of Mathematical Creativity in First Grade Students, (Part 1) (Chapters 1-4).
INSTITUTION	Wisconsin Univ., Madison. Research and Development Center for Cognitive Learning.
SPONS AGENCY	Office of Education (DHEW), Washington, D.C. Bureau of Research.
REPORT NO	TR-112
BUREAU NO	BR-5-0216
PUB DATE	Jan 70
CONTRACT	OEC-5-10-154
NOTE	92p.
EDRS PRICE	EDRS Price MF-$0.50 HC-$4.70
DESCRIPTORS	*Creative Thinking, Doctoral Theses, *Elementary School Mathematics, *Grade 1, *Learning, Mathematical Concepts, Matnematical Experience

ABSTRACT

Reported is the development and testing of a program designed to encourage individual creative mathematical activity in first grade students. Initially some characteristics of the creative process and creative thinking were examined and six criteria describing certain aspects of mathematical creativity were identified and validated. An instrument used to measure observable mathematical creativity was designed in order to test the program. An experiment was conducted to determine the effects of participation in this program on mathematical creativity. Two hypotheses were formulated: H1: "Participation in the program will increase a student's observable mathematical creativity"; H2: "Participation in the program will not affect a student's performance on a test of general creative ability". Par t I of this report includes Chapters I-IV. A statement of the reasons for the selection of this particular problem is presented in Chapter I. Chapters II-IV discuss the various considerations involved in designing and conducting the experiment. (FL)

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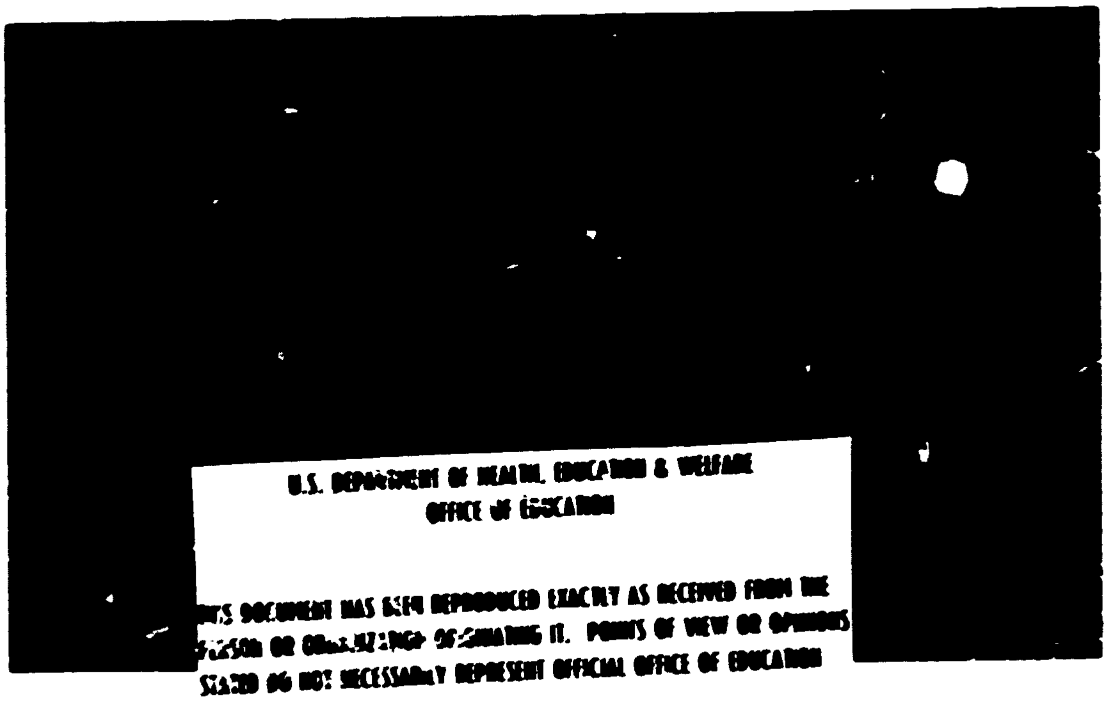

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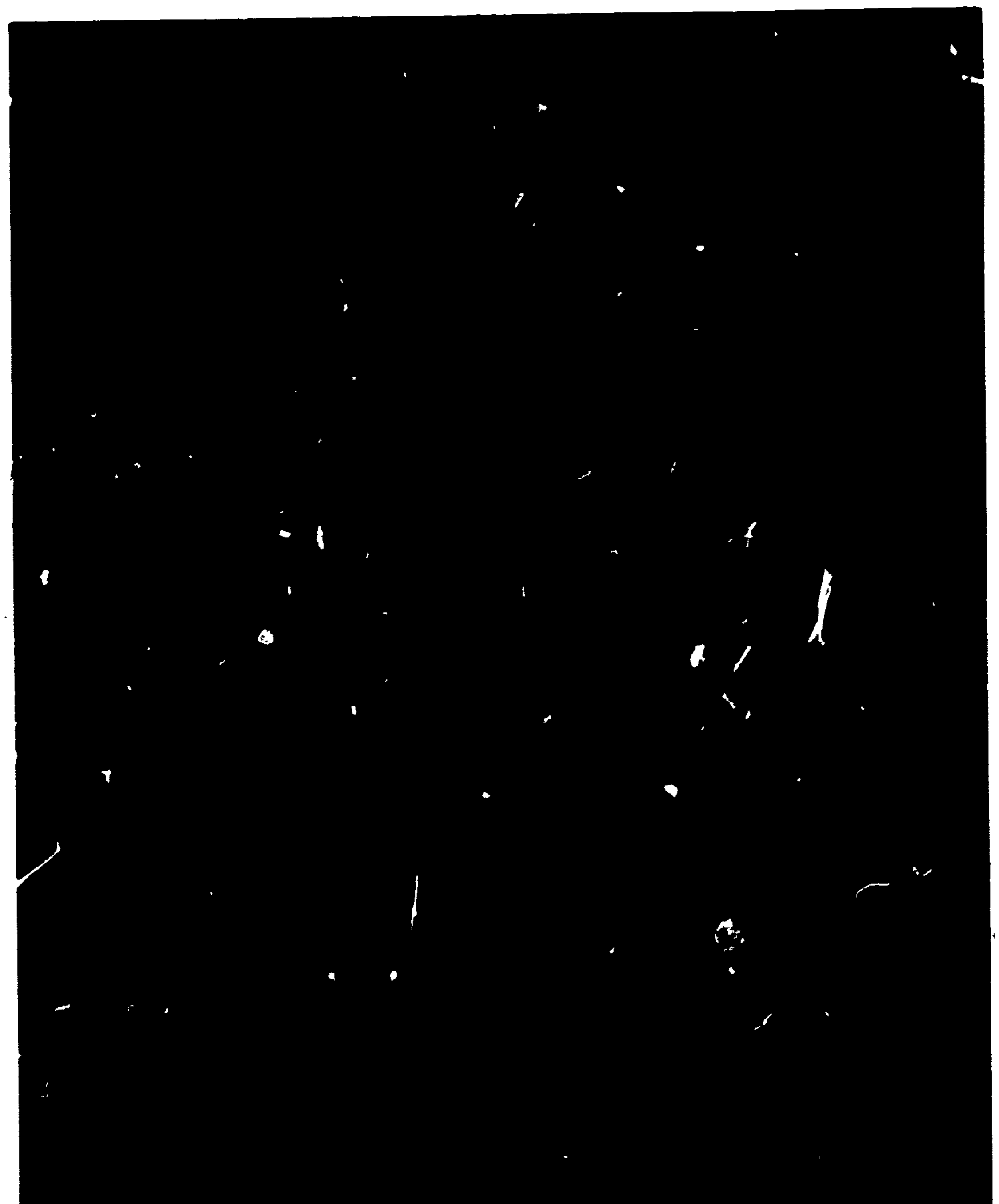
Report from the Project on
Analysis of Mathematics Instruction



U.S. Office of Education Center No. C-03 Contract OE 5-10-154

BR-5-0216
TR 112-Part I
OE/BR
PA-24

ED038292

Technical Report No. 112 (Part I) (Chapters I-IV)

THE IDENTIFICATION AND ENCOURAGEMENT OF MATHEMATICAL CREATIVITY IN FIRST GRADE STUDENTS

Report from the Project on
Analysis of Mathematics Instruction

By Rochelle Wilson Meyer

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Wisconsin Research and Development
Center for Cognitive Learning
The University of Wisconsin
Madison, Wisconsin

January 1970

This Technical Report is a doctoral dissertation reporting research supported by the Wisconsin Research and Development Center for Cognitive Learning. Since it has been approved by a University Examining Committee, it has not been reviewed by the Center. It is published by the Center as a record of some of the Center's activities and as a service to the student. The bound original is in The University of Wisconsin Memorial Library.

Published by the Wisconsin Research and Development Center for Cognitive Learning, supported in part as a research and development center by funds from the United States Office of Education, Department of Health, Education, and Welfare. The opinions expressed herein do not necessarily reflect the position or policy of the Office of Education and no official endorsement by the Office of Education should be inferred.

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STATEMENT OF FOCUS

The Wisconsin Research and Development Center for Cognitive Learning focuses on contributing to a better understanding of cognitive learning by children and youth and to the improvement of related educational practices. The strategy for research and development is comprehensive. It includes basic research to generate new knowledge about the conditions and processes of learning and about the processes of instruction, and the subsequent development of research-based instructional materials, many of which are designed for use by teachers and others for use by students. These materials are tested and refined in school settings. Throughout these operations behavioral scientists, curriculum experts, academic scholars, and school people interact, insuring that the results of Center activities are based soundly on knowledge of subject matter and cognitive learning and that they are applied to the improvement of educational practice.

This Technical Report is from Phase 2 of the Project on Prototypic Instructional Systems in Elementary Mathematics in Program 2. General objectives of the Program are to establish rationale and strategy for developing instructional systems, to identify sequences of concepts and cognitive skills, to develop assessment procedures for those concepts and skills, to identify or develop instructional materials associated with the concepts and cognitive skills, and to generate new knowledge about instructional procedures. Contributing to the Program objectives, the Mathematics Project, Phase 1, is developing and testing a televised course in arithmetic for Grades 1-6 which provides not only a complete program of instruction for the pupils but also inservice training for teachers. Phase 2 has a long-term goal of providing an individually guided instructional program in elementary mathematics. Preliminary activities include identifying instructional objectives, student activities, teacher activities materials, and assessment procedures for integration into a total mathematics curriculum. The third phase focuses on the development of a computer system for managing individually guided instruction in mathematics and on a later extension of the system's applicability.

To my parents

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ACKNOWLEDGEMENTS

Many people have helped in the research reported in this thesis. Although it is impossible to mention everyone here, some people deserve special recognition for their assistance.

Dr. John G. Harvey has been what good major professors are supposed to be--he has been of valuable help to me both in formulating the research and in writing the thesis. I was not once but twice fortunate; in addition to Dr. Harvey I was able to benefit from the advice of Dr. Thomas A. Romberg, who has given of his time and ideas as if I were his own student. These two, plus Dr. Joshua Chover and Dr. Gary A. Davis, read the drafts of the thesis and made many good suggestions.

Two pilot studies were graciously accepted into the Prospect Street Elementary School, Lake Mills, Wisconsin. For their cooperation I thank Mr. Ronald Hering, Director of Research Activities; Mr. Thomas Block, Principal; Mrs. Linda Bender, Mrs. Marianna Buchanan, Miss Cheryl Hagan, and Mrs. Inez Shultz, teachers. A second gracious acceptance allowed the experiment to be conducted in the Poynette Elementary School, Poynette, Wisconsin. I am indebted to Mr. Gerald Makie, Superintendent; Mr. Glenn Porterfiled, Principal; Mr. Charles Tucker, Elementary Supervisor; and Mrs. Hansen, the teacher, for their warm welcome and cooperation. Mr. George Glasrud located these two friendly school districts and made the initial contacts.

For taking time out of their busy schedules to participate in the face validation I thank Dr. Richard A. Brualdi, Dr. Joshua Chover, Dr. Donald Crowe, Dr. Simon Hellerstein, Dr. Mary Ellen Rudin, Dr. Hans Schneider, and Dr. Melvin C. Thornton.

Mr. John McFee did the videotaping and editing and managed to remain cheerful and interesting through it all.

Mrs. Carolyn Gornowicz did such a fine job of teaching during the experiment that I could relax and enjoy the students, knowing that they were in good hands.

Mr. Thomas Fischbach helped in some difficult problems of statistical analysis.

Mrs. Cindy Gaa not only typed the manuscript but pointed out missing verbs and commas along the way.

This research was supported by the United States Office of Education, Department of Health, Education, and Welfare, under the provisions of the Cooperative Research Program, through a contract with the Wisconsin Research and Development Center for Cognitive Learning (Center No. C-03/Contract OE 5-10-154).

My husband, Walter Meyer, has helped in ways that words cannot adequately express--ways that are in some sense so subtle that I find new aspects creeping into my awareness every day. All this while he too was writing a thesis.

And last, the students who took part in the pilot studies and the experiment and whose honest and creative reactions to the mathematics problems were wonderful and rewarding to watch.

ABSTRACT

The importance of mathematical creativity is widely acknowledged. The initial research was an examination of some characteristics of the creative process and the creative person. On the basis of this background, six criteria describing observable aspects of mathematical creativity were identified. These criteria were face validated by seven Professors of Mathematics at the University of Wisconsin and serve as part of a test instrument to measure observable mathematical creativity. One set of conditions conducive to mathematical creativity was proposed and activities which satisfy these conditions were piloted. From these activities both an instructional program to encourage individual mathematical creativity in first grade students and two problems to use a part of the test instrument were developed. An experiment was conducted to determine the effects of participation in the program on observable mathematical creativity; these effects were measured using the test instrument developed for this thesis. The effects on general creativity were measured using the Torrance Tests of Creative Thinking, Figural Forms A and B. The major contributions of this thesis are the identification and face validation of six criteria which describe observable aspects of mathematical creativity and the presentation of evidence that under suitable conditions first grade students can exhibit behaviors satisfying these criteria.

Chapter I

BACKGROUND

1.1 A BRIEF STATEMENT OF THE PROBLEM

This thesis reports the development and testing of a program to encourage individual creative mathematical activity in first-grade students. The effect of participation in the program on observable mathematical creativity and on general, non-subject-related, creativity was measured. The instrument used to measure observable mathematical creativity was developed in order to test the program; a record of its development is included in this thesis.

The working hypotheses were H1: Participation in the program will increase a student's observable mathematical creativity; and H2: Participation in the program will not affect a student's performance on a test of general creative ability.

1.2 OUTLINE OF THE THESIS

In Chapter I the reasons for the selection of this particular problem for study are presented and the background information on which the program and the test instrument were based is reviewed.

Chapters II, III, and IV are devoted to the various considerations which went into designing and conducting the experimental program.

The classroom conditions believed conducive to mathematical creativity, in terms of the problem activity and the actions of the teacher, are discussed in Chapter II. Two pilot studies were done to determine how those conditions should be interpreted in practice. These studies are reported and evaluated in Chapter III. From the activities used in the pilot studies, an experimental program was developed and conducted. The criteria by which some of those activities were selected for use in the program and a brief account of the program are in Chapter IV.

An experiment was designed to test the effects of the program on the participants. In order to measure the effect of the program on observable mathematical creativity, a test instrument had to be developed. A description of that instrument and the procedures by which it came into being are in Chapter V. The design of the experiment and choice of statistical analyses made on the data and the data are presented in Chapter VI. Conclusions and implications for futher research follow, in Chapter VII.

Four appendices follow Chapter VII. Appendix A is a journal of the experimental program. The next three appendices present some details of the test instrument: Appendix B contains the materials used for face validation of the criteria which are part of the test instrument; Appendix C contains a description of the mathematical problems used for the pretest and the posttest; Appendix D contains the materials used to score videotaped activities on the basis of the criteria.

1.3 OUTLINE OF CHAPTER I

A discussion of the importance of mathematical creativity begins the main body of Chapter I. The nature of general creativity follows, with particular attention given to the creative process and some of the personality traits associated with the creative person. Next, some techniques of mathematical creativity are compared with some techniques of general creativity. On the basis of what has been presented about mathematical creativity and general creativity, several researchable questions are posed. The problem researched in this thesis is an attempt to answer some of those questions. That problem and the means proposed to answer it are the topics of the final section of Chapter 1.

1.4 THE IMPORTANCE OF MATHEMATICAL CREATIVITY

The identification and encouragement of individual creative mathematical activity are of great importance to both the mathematician and the mathematics educator. An indication of how important can be inferred from Goals for School Mathematics in which one finds, in the section "Pedagogical Principles and Techniques," the subtitle "Fostering Independent and Creative Thinking," followed by a discussion of how the authors of that report felt this end might best be achieved. They strongly favored teacher directed discovery with the students working singly or in small groups. Such aids and innovations as a mathematics laboratory and a reference library of suitable extra projects are suggested. Also recommended is a restructuring of examinations to reflect the emphasis on understanding and creativity rather than responses which can be of a rote or mechanical nature (Cambridge Conference on School Mathematics 1963, pp. 17-20).

Surprisingly, the subject of mathematical creativity is not mentioned in the section of the book entitled "Broad goals of the School Mathematics Curriculum." It is not difficult to surmise that the eminent mathematicians and mathematics educators who participated in the Conference felt that the importance of identifying and encouraging independent and creative thinking was so great and so widely agreed upon as to not need stating.

In this thesis, something which a person produces will be called "creative" if it is derived from the combination of two or more ideas known to him in a way which is new to him. This meaning of "creative" puts the emphasis on the producer and conditions pertaining solely to him, and does not imply that the creative product is new in the context of all the world's knowledge. The word "creative" is often used in this way, which seems to reflect a value attached to the making of new combinations regardless of their ultimate utility. Perhaps this is because much importance has been attributed to the combining of two or more known ideas in a way which yields something new, since advances of knowledge in mathematics and other fields seen frequently to be from this source (Kaiser Aluminum and Chemical Corporation 1968, pp. 3-5).

Identification of creative mathematical activity is difficult because one wishes to credit not only creative combinations but also the kinds of activity which often lead to such combinations. In order to determine whether an effort to encourage creative mathematical activity has any effect, one needs some means of measuring differences in amount of creative mathematical activity. Constructing an instrument which could measure these differences is a generalization of the problem of identifying this activity, and thus must be based on decisions as to what kinds of activity are considered mathematically creative. In order to make these decisions it is necessary to examine the processes which often lead to creative combinations.

1.5 THE CREATIVE PROCESS: A SEQUENCE OF STAGES

The process by which a person produces something creative is not yet completely known, partly because an observer cannot have access to all the conscious thoughts of the person in question. Further, because the intervention of unconscious mental activity is a characteristic aspect of the creative act, it is highly likely that conscious processes would not be solely responsible for the creation.

A famous example of the intervention of unconscious mental activity is Poincaré's experience while stepping onto the bus at Coutances.

> Just at this time I left Caen, where I was then living, to go on a geologic excursion under the auspices of the school of mines. The changes of travel made me forget my mathematical work. Having reached Coutances, we entered an omnibus to go some place or other. At the moment when I put my foot on the step the idea came to me, without anything in my former thoughts seeming to have paved the way for it, that the transformations I had used to define the Fuchsian functions were identical with those of non-Euclidean geometry. I did not verify the idea; I should not have had time, as upon taking my seat in the omnibus, I went on with a conversation already commenced, but I felt a perfect certainty. On my return to Caen, for conscience's sake I verified the result at my leisure. (Poincaré 1913, pp. 387-388)

An interesting counterpoint to Poincaré's experience is that of another famous mathematician, Richard H. Bruck, who had a valuable illumination while stepping off a bus. Bruck had spent part of the summer and fall of 1947 working with incidence matrices of projective planes and orthogonal latin squares. During the winter of 1948, he sat on the final oral examination committee of Herbert Ryser, whose thesis concerned quadratic forms over the rationals. Bruck suspected that Ryser's ideas would have useful application to nets. Several meetings of the two produced little in that area. Then as Bruck was stepping off the bus which brought him home from work one day, he was struck by the idea that the proper application of Ryser's work was to incidence matrices of projective planes. This illumination led to the well-known Bruck-Ryser Theorem.*

Hadamard, who credited Helmholtz and Poincaré with first making the distinction, popularized the notion of four stages in the creative process: preparation, incubation, illumination, and verification (Hadamard 1954, p. 56). He cited as sources not only mathematicians but also physicists, chemists, poets, and musicians, among others. Other writers, including persons in all fields and psychologists interested in the creative process, have concurred with the general description given by Hadamard. It is of interest to note that the "creative people" whose introspective comments led to the formation of the notion of a "creative process" were generally those people considered creative by their peers because they were sustained producers of significant creative combinations (e.g. Hadamard 1954, p. 84).

*Richard H. Bruck, personal communication.

An elaboration of the four stages recently developed from various sources, which has been called "The Road to Creativity" lists seven stages: desire, preparation, manipulation, incubation, intimation, illumination, and verification. They are described as follows:

> DESIRE: The person must for some reason want to create something original. . . . For some reason, many people are simply not motivated, and with them the process never gets started.
>
> PREPARATION: . . . the process is analytical . . . 'making the strange familiar.'
>
> MANIPULATION: . . . an attempt at synthesis . . . 'make the familiar strange.'
>
> INCUBATION: . . . For reasons we do not fully understand, the unconscious mind keeps wrestling with the original problem . . .
>
> INTIMATION: . . . a feeling of premonition.
>
> ILLUMINATION: . . . 'insight' . . . the sudden release of strong psychic tensions.
>
> VERIFICATION: . . . the new pattern . . . is examined and valued . . . (Kaiser Aluminum and Chemical Corporation 1968, pp. 9-12).

Widespread agreement on the descriptive validity of the above models of creativity justifies the inference that when a person produces something creative--an illumination--he has experienced some previous creative mental activities--preparation, manipulation, incubation, intimation, illumination. It must be stressed that all the stages or phases in the models need not be identifiable in each instance of creativity. Intimation may not take place at all; an illumination may come during preparation or manipulation.

Given the inference that creative products imply creative mental activities, and because there is significant agreement among creative people in all fields as to a model of the creative process, one can assume with a reasonable degree of certainty that the implied creative mental activities are highly similar. This means that two creative results may differ in that one is worthless and one valuable viewed in the context of all human knowledge, or that the producer of one is a child and of the other an adult. Nevertheless the processes within each individual by which these creations are produced are close to identical. Because of the importance attributed to illuminations, differences in the age of the producer or the value of the illumination can be considered immaterial.

A better understanding of the creative process requires more details about each of the stages. There follow examinations of these stages ordered, with the exception of the stages which necessarily involve unconscious processes, in the manner opposite to normal occurrence.

Incubation, Intimation and Illumination

Little is known at the present time about how or why incubation, intimation, and illumination occur. Because of this, any attempt to encourage individual creativity must actively concentrate on the other four phases of the creative process. Allowance for incubation must be made in the timing of the attempt; for example spacing the attempt over a long period of time, rather than concentrating it into one or two days, seems desirable.

The making of a creative combination--an illumination--is often associated with mental processes which are not wholly conscious and is accompanied by a sudden release of tension and a feeling of pleasure and accomplishment. Precisely how and why this happens are open questions. It is agreed, however, that once having had this experience, a person is likely to anticipate its recurrence, and to work with renewed vigor. It is as if the creative process contains its own positive reinforcement.

Verification

At first glance one might be tempted to rule out the usefulness of verification in that it follows illumination in time and thus might have no relevance to the problem of whether or not there is an illumination. The models are sufficiently general, as has been noted previously, to include verification when in fact sometimes no attempt at verification is made or required. Verification is after all a measure of how well the illumination solves some problem. Some problems seem to require very little formal or time-consuming verification. Consider, for example, the truck which was too tall by an inch to drive under a bridge. A passing child suggested that some air be let out of the tires, thus lowering the truck sufficiently to allow it to pass. In this case, verification can be as small a task as understanding that letting air out of the tires will lower the truck. In some cases, like the arts, verification is not solely the province of the creator. Verification partially corresponds to a judgment of the worth of the creation made in the context of all human knowledge.

But the drudgery and possible disappointment of verification do serve a useful purpose. The very same efforts which can be labelled verification are often identical with preparation and manipulation corresponding to some future illumination. The person may or may not be aware of the extent that he is really pursuing two activities at once. In Hadamard, several famous creative people express the belief that illuminations are so often useful--that is, they survive verification--because of a "sense of fit and beauty" about his field which the creator has developed and which aids the unconscious selection of those combinations which become conscious illuminations (Hadamard 1954, pp. 29-32). The value of the verification stage in developing this sense of fit and beauty is obvious. It is well to note that it is the act of judgment of the creation made by the creator, not an observer, which aids in further creation.

The activities of preparation and manipulation can also contribute to one's sense of fit and beauty in his field. These stages are examined next.

Preparation and Manipulation

Preparation and manipulation have both local and global aspects. When posed a specific problem, a person examines the requirements which a solution must satisfy and pulls apart, perhaps reshuffles, the given elements. Often illumination comes not within the context of the specific problem but in the form of creating a new context in which to view the problem. This context-widening may involve combining the problem with something the person has not consciously thought of in days, months, or even years. Certainly the preparation and manipulation for such an illumination must encompass activities which took place over a long period of time. One can even view all of a person's life as preparation and manipulation for illuminations most of which will never happen.

"The Road to Creativity" equate preparation with "making the strange familiar" and manipulation with "making the familiar strange." A person cannot combine ideas unknown to him--this is fairly obvious. But what is just as important is that unless the familiar is viewed in a new way, it will not become a component of a creation. In part, the act of creation involves viewing an object or idea in such a way that its various aspects are isolated, separated from one another. Shape, color, function, movement, may all be aspects of an object, and as such are related. But if the function could be preserved while shape, color and movement were varied, then function is an isolatable aspect. To isolate an aspect can be likened to grabbing hold of it--hence the isolated aspect becomes a handle or a hook. Preparation could be called "the collecting of ideas" and manipulation "the attaching of hooks." This analogy was used by Poincaré.

> Figure the future elements of our combinations as something like the hooked atoms of Epicurus. During the complete repose of the mind, these atoms are motionless; they are, so to speak, hooked to the wall; so this complete rest may be indefinitely prolonged without the atoms meeting, and conse- without any combination between them. On the other hand, during a period of apparent rest and unconscious work, certain of them are detached from the wall and put in motion. They flash in every direction through space

> . . . like molecules of gas in the kinematic theory of gases. Then their mutual impacts may produce new combinations. . . . We think we have done no good, because we have moved these elements a thousand different ways in seeking to assemble them, and have found no satisfactory aggregate. But, after this shaking up imposed upon them by our will, these atoms do not return to their primitive rest. They freely continue their dance. (Poincaré 1913, p. 393)

The more ideas with more hooks a person knows, the greater his chances for illumination--the hooking together of two or more ideas.

Certain procedures seem to be directed to the goal of attaching hooks. Inquiry among creative people in many fields has revealed that the techniques followed by these people in approaching new situations are highly similar. Because the use of these techniques is positively correlated with the making of creative combinations, various instructional programs have been developed to teach people of different ages some of these approach procedures in a general, non-subject-related context (Crutchfield and Covington 1963, Davis 1969, Torrence 1967). A discussion of some of these techniques of general creativity and their relationship to mathematics is in section 1.7 MATHEMATICAL CREATIVITY: A SPECIAL CASE. Experiments have shown that participation in such a program does improve some of the so-called "creative abilities." Definitions of these abilities and a discussion of the way in which they were isolated are in section 1.6 THE CREATIVE PERSON. The tests used to measure general creativity do not, as a rule, evaluate the means by which the ideas were produced. These experiments in general creativity lend support to the hypothesis that a person's mathematical creativity could be increased through participation in a suitable program.

Before a person even begins to work on some problem, he must be motivated to do so. The focus now turns to the reasons a person would accept the challenge of a problem.

Desire or Motivation

Crutchfield distinguishes two kinds of desire or motivation which can be directed toward creativity. The first kind is task-involved or intrinsic to the problem at hand. Included here would be desire for a solution or for a good or elegant solution; desire to paint a picture or to make music (with pen or piccolo); desire to find out why or to learn about. The second kind of motivation is ego-involved or extrinsic to the problem. This kind of desire, for example the desire for status, for self-enhancement, or for a good grade, puts the solution of the problem at hand in the position of a means, subordinate to some external end. This is true whether or not the activity involved in solving the problem has a creative completion (Crutchfield 1967, pp. 121-122).

Intense desire to be creative can hamper creativity. Crutchfield suggests that ". . . the person's seeking to achieve 'creativity' _per se_ as his goal . . . is merely a form of ego-need . . ." He continues:

> The crucial significance of the distinction between ego-involved and task-involved motivations for the creative act can be expressed in the hypothesis that the quantity and quality of creative acts will, in general, be higher under conditions of task-involvement than under condtitions of ego-involvement (Crutchfield 1967, pp. 122-123).

In order to create, writes Crutchfield, a person must first free himself from old ways of thought and then produce original insights.

Laboratory and clinical evidence indicates to that author that intense ego-involvement, as compared with task involvement, is detrimental to cognitive functioning, especially when such functioning requires putting aside habitual modes of perception and thought. In addition, it often seems to the person who gets an insight that the idea emerged spontaneously and could not be forced. Perhaps intense ego-involvement is thus detrimental to the preparation and manipulation stages and also to the process of illumination (Crutchfield 1967, p. 124).

The desire phase of creativity, in the light of the Crutchfield essay, should refer only to intrinsic or task-involved desire. Exhortations to be creative, imaginative, original, (it matters not what adjective is used) can be detrimental to creative thinking because they can arouse ego-involved motivations. A person will be more creative within a situation if he is interested in that situation for what is intrinsic to it.

In order to learn what factors influence the kind of motivation a person has with respect to a problem, the focus now turns to the creative person.

1.6 THE CREATIVE PERSON

When a group of people have been characterized on the basis of one attribute, for example being creative, it is always interesting to look for other attributes which the members of the group might share. It happens that creative people share many characteristics; and there seem to be indications that certain of these characteristics are prerequisites for creative activity because of the possible conflicts between the creative person and the society in which he lives.

Socialization vs. Creation: A Source of Conflict

Man is by nature an organizer. He structures and classifies elements of his environment according to their similarities and differences. The fact that all human cultures include laws, ranging from exogamy to alimony, gives evidence of this activity of man.

Socialization is the means by which a society passes on its culture to the young. This process involves the transmission of the culture's organizations and classifications to those who might, if left to their own devices, structure their environment in an unorthodox way. School is, of course, one means of socialization.

Crutchfield states, "In brief, the creative solution involves the emergence of a new and 'fitting' organization of the elements of the problem," (Crutchfield 1967, p. 123). Creative solutions are then a counterforce to socialization to the extent that the culture does not value new organizations. Or alternatively, societies are to the same extent anti-creative. Comparison of the Dark Ages with the Golden Age of Greece on the degree to which new structures were valued and the contributions to man's knowledge from each era lends support to this. Within a society there are some areas where creativity is encouraged and some where it is discouraged. Technological advances, such as making cars which do not pollute the air, are encouraged in the United States. On the other hand, there would probably be great initial resistance to a proposal to replace private transportation with public transportation, no matter how convenient and non-polluting it were.

The existence of a culture means that there are barriers protecting many elements from change. A creative person might be consciously attacking barriers, despite restraining influences. One would therefore expect that the creative person is in some way insulated from these restraining influences.

When a person's judgment is in conflict with the judgment of a group of which he is a member, and when the person expresses agreement with the group judgment, one says he is conforming. Crutchfield writes, ". . . conformity pressures may be expected to be injurious to creative thinking . . . because . . . outer pressure and inner compulsion to comform arouse extrinsic, ego involved motives in the problem solver." In addition, ". . . persons who are expecially susceptible to conformity pressures. . . tend to have other personality characteristics that are deleterious for creative thinking." (Crutchfield 1967, p. 125)

The above remarks are from the beginning of the Crutchfield article; the expected negative correlation between the tendency to conform and successful attempts at creative thinking is subsequently supported by experimental evidence. A standard measurement technique, administered to various groups of individuals in various experiments, yields a conformity score, which is the percentage of a standard set of 12 objective-type items on which the person yielded to the false answer of what he felt was a group of his peers. These scores were higher for groups whose members were considered by their peers low in "creativity" and "originality" and were higher in some professions, for example military officers, and lower in others, for example research scientists in industry.

Personality Traits Correlated with Creativity

The tendency not to conform to group pressure is but one trait frequently found in creative people. One profile of a creative person suggests some other personality characteristics generally correlated with creativity:

> INHERITED SENSITIVITY-- . . . greater sensitivity to certain types of experience. . .
>
> EARLY TRAINING-- . . . childhood in a home atmosphere that encouraged rather than discouraged, inquisitiveness. . .
>
> LIBERAL EDUCATION-- . . . teachers and curricula that place a premium upon questions . . . reward curiosity. . .
>
> ASYMMETRICAL WAYS OF THOUGHT-- . . . the creative person finds an original kind of order in disorder. . .
>
> PERSONAL COURAGE-- . . . not afraid of failure, or of being laughed at. . .
>
> SUSTAINED CURIOSITY-- . . . never stops asking questions, even of his most cherished ideas . . . capacity for childlike wonder. . .
>
> NOT TIME-BOUND-- . . . does not work by the clock . . .
>
> DEDICATION-- . . . problem will not be left unsolved. . .
>
> WILLINGNESS TO WORK-- . . . may not express itself in the number of hours . . . but in the fact that even in sleep or reverie the creative person is constantly working for a solution. . . (Kaiser Aluminum and Chemical Corporation 1968, p. 24)

Davis has described creativity in terms of three major parameters or dimensions: attitudes, abilities, and techniques (Davis 1969, p. 539). Techniques are methods of preparing and manipulating, and have been partially discussed in section 1.5, THE CREATIVE PROCESS. The above profile can be profitably examined using the remaining two Davis dimensions--attitudes and abilities.

The above profile gives a partial list of attitudes associated with creative people: not afraid of failure or ridicule, inner-directed, inquisitive, flexible in beliefs, capable of childlike wonder. Existing programs for encouraging general creativity attempt to impart some of these attitudes to the students. The early training and liberal education characteristics cited in the profile are forces which are credited with resulting in the attitudes listed above. Any program, in attempting to influence attitudes, is really trying to add a creativity-encouraging force to the forces already existing in the person's life.

The importance of positive attitudes can not be overemphasized. One group creativity technique, brainstorming, developed by Osborn, puts most of its emphasis on factors affecting attitude, such as deferring criticism and acceptance of all ideas, no matter how wild. The methods by which ideas are produced are not as specified, although combination and improvement of others' ideas are sought (Osborn 1963, p. 56).

In summary, it seems that like other aspects of personality, a proper creative attitude flourishes best when early training and continued experiences encourage it.

The above profile seems to list only two abilities: asymmetrical ways of thought and sensitivity, and latter assumed to be inherited. This raises the old and as yet unresolved controversy of the roles of heredity and environment. If on the one hand, creativity is a learned response, then a program to promote creativity is actually teaching it. On the other hand, creating may be a natural human activity, limited by some inherited ceiling called ability and be circumstances of the environment. In this view, a program to encourage creativity is an attempt to set up optimal conditions under which creativity may occur. The proper setting is a necessity in either view; conditions which provide the right environment will be discussed in Chapter II.

The profile was compiled from several sources most of which derived their information from interviews and introspective comments of creative people. Another way to isolate those abilities which distinguish creative people is through factor analysis. This method has been used by Guilford, who has identified certain abilities which may bear directly on creativity. These include fluency, the number of ideas produced; flexibility, the number of categories into which the ideas fall; originality, the unusualness of the ideas; and elaboration, the number of ideas added to the basic one (Guilford 1962, pp. 156-160). Some of the tests which are used to measure general creativity attempt to measure these abilities (Torrance 1966, p. 9).

A Particular Correlate: Nonverbal Thought

Hadamard discusses a charactersitic of mental activity which may be a correlate of high creative ability, especially among persons who work in areas which involve a low degree of communication in everyday language. This correlate is the use of personal symbols and images, as opposed to words, in thought. Those with such a turn of mind find that words are aften deceptive, and mask the true nature of the ideas under consideration. The disposition is not without drawback:

> 'It is,' he [Francis Galton] says, 'a serious drawback to me in writing, and still more in explaining myself, that I do not so easily think in words as otherwise. It often happens that after being hard at work, and having arrived at results that are perfectly clear and satisfactory to myself, when I try to express them in language I feel that I must begin by putting myself upon quite another intellectual plane. I have to translate my thoughts into a language that does not run very evenly with them. I therefore waste a vast deal of time in seeking for appropriate words and phrases, and am conscious, when required to speak on a sudden, or being often very obscure through mere verbal maladroitness, and not through want of clearness of perception. This is one of the small annoyances of my life.' (Hadamard 1954, p. 69)

The interrelation of language and thought is an interesting and open question. There seems to be agreement that young children do use personal symbols. Piaget believes that the child must develop a personal conception before he can correctly use the words which denote that concept. He does not seem to be interested in the role played by the use of words in the child's environment in helping the child develop his personal conception (Flavell 1963, pp. 270-275). Vygotsky is primarily interested in this latter. He believes that thought, or inner speech, is an outgrowth of social speech, and that social symbols become personalized (Vygotsky 1962, pp. 9-24).

Whatever the cause, the phenomenon is certain--children, as well as some adults, may encounter great difficulty expressing their thoughts verbally, and the attempt to verbalize may inhibit thought process (Hendrix 1961, p. 292). Therefore, anyone who wishes to identify or encourage creative activity in another person must allow for non-verbal behaviors.

1.7 MATHEMATICAL CREATIVITY: A SPECIAL CASE

Techniques of Mathematics and General Creativity: A Partial Comparison

It would be impossible to list all the techniques which have been used to arrive at creative products. Just those techniques taught in general creativity training programs would make an impressive list. As examples, one could take the four techniques considered by Davis: attribute listing, morphological-synthesis, checklisting, and synectics (Davis 1969, pp. 540-543).

Attribute listing is a simple and straightforward method for generating ideas in which one itemizes the important attributes of something and then considers each attribute as a source of potential change or improvement. Unfortunately, the important attributes of mathematical proofs include not just hypotheses and conclusions but more crucially the functions and interrelations of the hypothesized elements. The nature of mathematics is such that the difficulty is in pinpointing the contribution of each element of the hypothesis. The method of attribute listing was developed for problems whose important attributes are easily identifiable, and it seems to be limited to situations having that characteristic.

There are a few cases in mathematics where the second phase of attribute listing seems to have been used in the examination of axion systems, where the relevant attributes, the axioms themselves, have already been listed. Examples of this are non-Euclidean geometries and Extraordinary Cohomology Theory.

Morphological-synthesis is a highly structured technique somewhat related to attribute listing. A few basic characteristics of the problem are chosen and a list of specific values for each is made. Then one systematically examines all possible combinations of one value of each attribute. It is not clear whether this technique has ever been used in mathematics. This author suspects that in order to produce any worthwhile mathematics through use of this highly structured technique, very careful and creative choices of characteristics and specific values would be necessary.

A checklist in mathematics could be useful. One could list the techniques found successful in previous problems and then try each one, or combinations of them on the problem at hand. As with any checklist, not every item on the list is universally applicable and judgment must be used. A checklist is not a technique or process, but an aid; its use does not necessarily impose a structure to the attack on the problem. Most mathematicians do use this checklist procedure in an informal way. An excellent example of a mathematician's checklist is the outline of problem attack by Polya in How To Solve It (Polya 1957, pp. xvi-xvii). The major divisions of that outline are "Understanding the problem," "Divising a plan," "Carrying out the plan," and "Looking back." Each is elaborated with leading questions and explanatory remarks.

Much of the elaboration in the Polya outline resembles the kind of problem analysis used in the synectic method. This technique relies on use of metaphor and simile. Analogies are sought between the problem and seemingly unrelated areas in which relationships hold which are similar to those in the problem. The method is open-ended and unstructured. It encourages speculation about function and postulating ideal situations. Under "Devising a plan," Polya suggests looking for or recalling a problem related to the one at hand and trying to apply the result or method of that related problem. "Related" is elaborated as "more general," "more special," "analogous," and "having same or similar unknown." The mathematician's concept of ideal is implicit in the question, "Could you change the unknown [conclusion] or the data [hypotheses], or both if necessary, so that the new unknown and the new data are nearer to each other?" (Polya 1957, pp. xvi-xvii).

Some of the techniques of general creativity are normally used in mathematics, some are not. The more open-ended, less structured methods seem the more common in mathematics.

Some Unanswered Questions

Much is now known about creativity. A description of the creative process has been widely agreed upon, and some of the stages of that process have been examined in detail. The creative person has been characterized by a range of attributes in addition to his creative activity. On the basis of what has been learned, new and often more valuable questions can be framed. Some of these questions follow.

Precisely what role does the unconscious play in the creative process? How does this "sense of fit and beauty" aid the unconscious in its role?

Are the personality correlates of creative ability particular to this time or to Western cultures?

Since general creativity programs are known to increase scores on tests measuring the ideas produced on several creative ability rankings but do not evaluate the means by which the ideas were produced, are these programs influencing the techniques used or the attitude toward unusual ideas? Does it matter?

The entire relationship between mathematical creativity and general creativity is unclear in terms of both techniques and abilities. In particular, one could ask: Would participation in a general creativity training program increase one's mathematical creativity; would participation in a program teaching mathematically relevant creative approaches improve one's performance on a test of general creativity? And more important to mathematicians and mathematics educators is the question: Would participation in a program teaching mathematically relevant creative approaches increase one's mathematical creativity?

The Problem Restated

From the preceding discussions and questions, several researchable problems can be drawn. One will be posed here.

It has been noted that general crativity has been successfully encouraged and that mathematical creativity is important. It is natural to begin developing means to encourage individual mathematical creativity. This immediately presents certain questions: whose mathematical creativity and how is this encouraging to be done. And of course, the problem of measuring the effects of this encouragement must be faced.

Several considerations help answer the question "Who?". First, it has been indicated that early and continuing positive experiences at exercising one's creative abilities seems to be of great benefit. These experiences are credited with helping establish a favorable attitude towards creative ideas and with developing in the person a desire to seek a favorable climate for creative activity. The second consideration is the nature of creativity. Because the creative process is so similar whenever and in whomever it occurs, an attempt to encourage individual mathematical creativity is not limited to persons of a certain age; in particular, encouraging early experiences is not an unreasonable goal. The third consideration is where should the attempt take place. It is natural to desire to see mathematical creativity identified and encouraged as a part of the mathematics curriculum in the schools. This would, among other things, add a pro-creativity force to the scholastic aspect of the socialization process, which can as a whole be rather anti-creative. These three considerations lead to one conclusion: as a start, focus on early primary children, those in kindergarten or first grade.

If students in the early primary years can be shown capable of mathematical creativity, then the circumstances under which this creativity was demonstrated could be examined and generalized to provide guidelines for constructing circumstances under which students with more mathematical sophistication could also be mathematically creative. By showing the possibility of early experiences, the groundwork for the continuation of those experiences is laid.

It is by no means obvious that students so young are capable of mathematical creativity. They know almost no formal mathematics. They tend to be less verbally fluent than older students. And they operate in a concrete, not abstract, fashion. That is, in order for them to understand an abstract principle, it must first be presented in many concrete embodiments and dealt with in that way (Piaget 1964, pp. 9-10).

On the other hand, children at this age have certain assets precisely because they "know" so little. Knowledge often takes the form of adopting ready-made structures for organizing facts or experience. Because creativity involves restructuring, one might expect that chances for this restructuring are greater from those who have not been strongly and regularly reinforced into thinking in terms of these ready-made structures. In addition, young children have the capacity to wonder, the inquisitiveness, which is so rare in adults and is one of the traits correlated with creative adults.

The specific problem researched had several parts: Can first-grade students demonstrate any observable mathematical creativity? Can the amount of this observable mathematical creativity be increased through participation in a program combining appropriate mathematical problems and suitable instruction? What would be the effect of participation in this program on general creativity?

The means used to answer these questions is the development of a program to encourage individual creative mathematical activity in first-grade students and the measurement of the effect of participation in the program on observable mathematical creativity and on general creativity.

Since the program must focus on encouraging and positively reinforcing creative mathematical activity and attitudes favorable to mathematical creativity, one hypothesis is H1: Participation in the program will increase a student's observable mathematical creativity. Transfer of attitudes and approach from mathematical creativity to general creativity would be desirable. However, in order for the program participants to transfer the attitudes and approach, they must perceive the two situations as similar in some way (Klausmeier and Goodwin 1966, p. 475). Because no attempt was to be made to indicate to the students that such transfers could be made and few general principles were to be stated during the program, it was expected that transfer will not occur. Consequently H2: Participation in the program will not affect a student's performance on a test of general creative ability.

In order to test these hypotheses, an experiment was conducted. A test instrument was designed to measure observable mathematical creativity. This instrument has two parts: criteria and problem. A student working on a mathematical problem suited to his background and in the case of the experiment, the content of the instructional program, is observed and his behavior scored on the basis of six criteria. Each of the criteria describes an aspect of creative mathematical activity in terms of observable behaviors. Observable behaviors were used because the reliability of criteria so written is higher and because the problem of whether one person can know what another person is thinking can in this way be circumvented. For the purposes of the experiment, the observations were made via videotape in order that the presence of observers would not upset the students and that the observers would not know whether they were scoring pretests or posttests. This test instrument is discussed in more detail in Chapter V.

The effect of the program on general creativity was measured by the Torrance Tests of Creative Thinking, Figural Forms A and B (Torrance 1966a and 1966b).

The students chosen to participate in the experiment, both those in the program group, and those in the control group, were all members of one first-grade classroom. They were randomly selected from one of three groups, stratified on the basis of mathematics achievement. This was done for two reasons. First, general creativity, and perhaps mathematical creativity, is found distributed among all levels of IQ and scholastic achievement. (Guilford 1962, pp. 163-4) Second, it seems to be indicated by the very nature of the creative process that trying to encourage mathematical creativity in the child with less mathematical promise may provide that child with a rewarding mathematical experience which would invigorate his interest in this subject.

Summary

An examination of the present knowledge of creativity has led to several questions, one of which is of particular importance to mathematicians and mathematics educators. That question is, "Can participation in a program combining appropriate mathematical problems and suitable instruction increase one's observable mathematical creativity?" In order to obtain an answer to the question, such a program was developed and tested. The interpretation of the phrases "appropriate mathematical problems" and "suitable instruction" so that the program would be properly suited to a first-grade student was a nontrivial task. The next two chapters report the progress of that interpretation, first in theory, then in pilot studies.

Chapter II

CLASSROOM CONDITIONS WHICH ENCOURAGE MATHEMATICAL CREATIVITY

2.1 OUTLINE OF CHAPTER II

This chapter describes that particular variation of teacher-directed discovery which seems best suited to encouraging mathematical creativity. The specific aspects of the classroom conditions which are considered are first, the characteristics of the mathematical situations, and second, the actions of the teacher in presenting the situation and while the students are pursuing the activity. A third condition, the length and spacing of the attempts to encourage mathematical creativity, is considered in the last section of this chapter.

2.2 THE MATHEMATICAL SITUATION

Certain characteristics of the mathematical situations to be used to encourage mathematically creative activity are rather straightforward. The situations must be mathematical in content (specific details of this requirement are presented later in this section) and must be suitable to the students' level of sophistication and dexterity. Based on the information about the manner in which young people seem to learn discussed in the previous chapter, the latter requirement can be elaborated as follows in the case of first grade students:

The mathematical content should be embodied in concrete materials and actions and the manipulation of the materials in a mathematically relevant way should not depend on a great precision of action. The author assumes that first grade students are able to count to 25 and to compare integers or number of discrete objects to a maximum of 25. Under this assumption, mathematical situations are considered appropriate to the students' level of sophistication if, on the basis of informal analysis, they seem not to be based on mathematical abilities other than these.

According to Smith, a most crucial requirement of situations which encourage creativity is that they be open-ended; that is, in the case of mathematical creativity, that they be problems to which several possible mathematically appropriate responses, ideally at different levels, could be made. This means that the exploration of possibilities and the choice of several possible goals, both examples of divergent thinking processes, will be necessary before any problem solving, or convergent thinking, could be begun. Smith maintains that the existence of a problem acts as motivational tension; a solution of the problem brings to the solver a relief of tension and a feeling of pleasure and accomplishment (Smith 1966, p. 157). It is therefore important that the content and structure of the problem be such that every student could produce a solution.

One difference between an open-ended situation and a sequence of problems leading up to a specific "discovery" is that each student can make a "right" discovery or response to an open ended situation because there is no one end to which all activities lead. In an open-ended problem situation, as Smith describes it, ideas or objects are manipulated and explored, and what results from the activity is often new, different, and unpredictable. Each student should be able to experience some degree of success in his encounter with the situation since no one solution is "right" or "best" by authority of the teacher. This is part of what is meant when it is said that conditions conducive to creativity are success-oriented (Smith 1966, pp. 157-162).

Open-ended does not mean without structure. It has been found that if the activities in an elementary classroom have very much or very little structure generally little creativity results; a moderate amount of structure seems to be called for (Westcott and Smith 1967, p. 15). The interpretation of this requirement in practice is not easy; the report of the two pilot studies, in Chapter III, gives the details of how the author tried to meet this condition.

Because an important aspect of the program will be encouraging creative techniques of approaching a situation, situations which would tend to generate observable behaviors, not just an end product or "answer," are preferred. This allows the teacher to follow the progress of the student's work without asking for verbal responses and without otherwise interfering.

In order to minimize the possible effect of previous mathematical achievement on successful experiences in the program, the author required that the situations for the program have primarily geometric rather than arithmetic content. Since few first grade students have had much formal exposure to geometry, it was expected that all of the participating students would begin the activities in the experimental program with non-negative attitudes toward the content and with about the same degree of preparation.

Thus, one kind of mathematics situation which seems well suited to encouraging the creative activity of first grade students are open-ended geometric problems having a moderate amount of structure, using concrete materials but not depending on a great precision of action, seeming to have no mathematical prerequisites other than counting and comparing integers or objects to 25, and generating observable behaviors during the process of solution.

2.3 THE ACTIONS OF THE TEACHER

The best of mathematical situations could fail to encourage any mathematical creativity if the conditions under which the situation is presented and under which the student has to work are unfavorable. The teacher has the primary responsibility for establishing and maintaining a climate which rewards, or at least does not punish creative efforts. This is in part true because the teacher sets the emotional tone of the classroom by his actions. The important role played by the attitude of the student towards mathematical creativity makes it imperative that the teacher be conscious of the great influence his actions and attitudes have on those of his students. It has been shown, for example, that students of highly creative teachers, who presumably have creative attitudes, produce more creative products than students of less creative teachers (Torrance 1962, pp. 71-72). In the "Ten Commandments for Teachers" given by Polya, the first commandment can be interpreted as a demand that mathematics teachers be mathematically creative to some extent; that commandment is "Be interested in your subject" (Polya 1965, p. 116). The research and opinion just cited are the primary reasons that the author specified a teacher-directed instructional program with a teacher who can enjoy being mathematically creative at some level, not necessarily a sophisticated one, and who realizes that in mathematical creativity, the process is as important as the product.

When the teacher presents an open ended problem to the students, he must be careful to stress by his actions and tone all of the aspects of the problem which are designed to encourage creativity. Smith suggests several ways in which the teacher can do this. He should say that he hopes the student will enjoy the activity. He should make clear what the guidelines or rules for the activity are, but he should equally stress that anything which follows those rules will be a good solution to the problem; there will be no extra conditions added later. The teacher should point out that every student could produce a different, but acceptable, solution to the problem and that there is no one "right answer" but many acceptable ways of solving the problem, and that he would like to see as many original and new solutions as the students can produce (Smith, 1966, pp. 157-162). The author would add to the suggestions of Smith that the introduction should be as brief as possible, so that the students do not lose the interest, which the teacher so carefully tries to build, before they get a chance to begin work.

Once the activity has been introduced and the students begin work, in at least one sense, the students and the teacher exchange roles. During the introduction, the teacher's primary role is to present information to the students; the students' primary role is to try to understand that information. When the students begin work, they become the primary sources of new information out of which the teacher tries to make sense in order that he can gain some understanding of what the student is trying to do. After the teacher introduces the class to an open-ended problem, each student starts to attack the problem activity in his own way. The teacher can subtly help and direct these attacks through praise and encouragement, so that each student will be able to produce his own solution, satisfying to him, within the framework of the problem. But in order to insure that the results of all the activity directed towards the problem could be something new, different, and perhaps unique, the teacher must first learn from each student what his goals are; then he can help him to achieve his goals. He must, as Polya suggests, put himself in the student's place (Polya 1965, p. 116).

The teacher can gain an understanding of what the student is doing through two means: he can observe the actions of the student and he can listen to what the student says. Observation is sometimes a more difficult method than listening, but it has definite advantages. Many students, especially young ones, cannot easily express themselves verbally. The attentions and questions of the teacher may harm the student if he feels that he is expected to explain in words what he is doing (Hendrix 1961, p. 292). The author feels that the teacher should accept as a guideline for his actions that it is better for him not to gain an understanding of what the student is attempting to do if in order to gain that understanding he risks interfering with and perhaps destroying the student's patterns of thought.

Underlying many of the principles basic to creative teaching which Smith proposes, and perhaps the most important aspect of the teacher's actions during the time when the students are working, is his acceptance of any solution or partial solution which lies within the framework of the problem or offers an interesting restructuring of the problem. This acceptance seems to serve several purposes.

The Crutchfield article discussed in the previous chapter presents several arguments based on experimental evidence which would tend to indicate that intrinsic motivation is more conducive to creative efforts than extrinsic motivation (Crutchfield 1967). If a student works on a problem becasue the teacher tells him to do so, his motivation toward the problem can be largely extrinsic to the problem. The greater the choice the student has in determining the problem he works on, the more likely it is that his motivation will be directly related to elements intrinsic to that problem. The teacher, by accepting the student's choice of a problem, recognizes the importance of intrinsic motivation and allows it to continue when it exists.

A second purpose served by the teacher's accepting attitude and actions is the formation of similar attitudes and actions in his students. A student may be hesitant to give outward evidence of having new or different ideas if he has learned that the rest of the class will ridicule him for having such ideas. But if the teacher encourages such ideas by accepting them, and by occasionally praising them, the students in a normal classroom will begin to mimic the teacher and become less hostile to new ideas. Thus the classroom atmosphere can be made more conducive to creativity.

The individual student who offers his ideas to the teacher may have put some creative thought into developing those ideas. If the teacher does not accept these offerings, his actions could be interpreted by the student as a punishment for his creative efforts. The third purpose served by the teacher's acceptance of all ideas is that this acceptance can act as positive reinforcement for individual creative efforts.

The accepting attitude of the teacher also helps the teacher to learn what the student is doing. Since the teacher does not evaluate or criticize what the student offers, his mind can concentrate more fully on the content presented. This aids the teacher in understanding the student's goals and the means he has chosen to reach them.

Acceptance of ideas is important, but it is not enough. Smith suggests that the teacher must emphasize the production of new and different ideas and should encourage the kinds of activities which often produce original ideas. The teacher can do this through praise and encouragement (Smith 1966, pp. 157-159).

Praise is a form of positive reinforcement. It is characterized by words such as "excellent" or "very interesting," which indicate the value of the ideas, actions, or results referred to. It is sometimes difficult to distinguish between praise and acceptance; praise must be verbal, acceptance need not be. Also, each teacher reserves certain words for praise and uses other words in a more neutral manner. Thus the word "good" could be an indication of acceptance if the teacher used it to comment on every student suggestion, or it could be reserved for occasional use by a different teacher and become a word of praise.

The author believes that praise must be used carefully by the teacher, since overuse may cause the student to become so interested in the reaction of the teacher that he pursues the activity as a means to getting positive reinforcement and not because he is interested in the task itself. One caution that the teacher should observe is that he should praise good and appropriate mathematical questions, investigations, conjectures, generalizations, modifications and results, but he should not praise the child for having produced these things. An idea may be praise-worthy because of its uniqueness, but the child offering the idea should not be called "clever" or "good" for producing the idea, if only because he would not be "stupid" or "bad" if he had not produced it. In addition, the teacher should rely primarily on his acceptance and encouragement to act as partial praise, and should directly praise only exceptionally good, useful, or unusual ideas.

Those kinds of actions which the teacher sometimes praises should always be encouraged by him. In addition, Polya suggests that the

the teacher encourage the student to ask questions, to make guesses, to look for patterns, and to look for something familiar (Polya 1965, pp. 116-120). Smith recommends that the teacher encourage the student to look for new uses, for modifications, for extensions, for substitutions, and for rearrangements. He also believes that the teacher should try to encourage the student's efforts to be different, unique, individual, and original (Smith 1965, pp. 159-165). One way in which the teacher can encourage these activities on the part of the student is by asking specific questions such as, "Do you have any more ideas about this?" or "Does this remind you of something we have seen before?". The teacher also encourages when he shows an interest in what the student is doing, especially when he shows that interest when the student is at a difficult and discouraging point in his work. Sometimes all the student really might need would be an extra hand so that he could build the next part of a structure; if the teacher can lend that hand and then move on, possible frustration can often be turned to the pleasure of accomplishment.

Sometimes a student needs specific mathematical help in order to complete a problem. At such times, the teacher should follow the advice of Polya: when a mathematics teacher gives help to a student, it is preferable for the help to be in the form of a general statement or suggestion which may be useful in solving problems in addition to the specific problem facing the student (Polya 1965, pp. 118-119). For example, if it is necessary for the student to undo some of his work and reconstruct it, the teacher should say something like, "Sometimes you have to undo what you have done and start again to get what you want," rather than trying to focus the student's attention on a particular element of his solution which sould be changed.

When a student becomes engrossed in a problem, he can lose sight of the original requirements of the problem and forget to evaluate his solution, that is to verify that his results or ideas satisfy the requirements. Verification is a necessary part of the creative process, as was discussed in section 1.5, **THE CREATIVE PROCESS**, so the teacher should try to ascertain whether the student is omitting it and remind him not to omit verification if he needs to be reminded. Often all that is needed is the teacher asking the student if he can recall the original requirements of the problem. Verification, as was seen in the previous chapter, is best done by the student himself, and the recalling of the original requirements often leads the student directly to verification. Sometimes the teacher participates in verification; both Polya and Smith insist that he do so only in the spirit of a helper in a constructive evaluation and must remember to emphasize the successful aspects of the thing being verified (Polya 1965, pp. 119-120; Smith 1966, pp. 161-162). The teacher should let the student find his own mistakes, but can help him by requesting that the student show why something is true or why it works. In order to prevent this request from becoming a tip-off to the student that womething is wrong, and perhaps cause him anxiety, the teacher should sometimes request proof of things which are right, or true, as well as those which are not.

Polya refers to his ten commandments as rules which embody aspects of the principle of active learning. As an additional guideline for mathematics teachers, he states the principle again, with emphasis on the teacher putting himself in the student's place: "Let your students ask questions; or ask such questions as they may ask by themselves. Let your students give the answers; or give such answers as they may give by themselves." The author feels that these guidelines are especially important when the goal of the mathematics lesson is that the students experience aspects of mathematical creativity.

In summary, the teacher should be aware of the effect on the student of his own actions and requests and should give preference to the student's progress and thoughts over his own understanding, especially in the area of verbal communication.

The teacher should have an accepting attitude, should encourage the students in the various aspects of creative mathematical activity, should occasionally praise the efforts of the student, should make sure that the student evaluates his work, and should in all of these activities try to put himself in the student's place.

2.4 THE DURATION AND SPACING OF THE LESSONS

Ideally, once a teacher has presented the students with a problem on which they can do some creative thinking, the duration and spacing of work on the problem should be the choice of each student. This ideal can be achieved somewhat in a student-structured classroom, but is, of course, impossible for a program which is to be part of an experiment and taught by someone other than the regular classroom teacher.

In order to allow time for incubation to occur, short daily lessons, rather than a few longer sessions were planned. It is possible that two or three lessons per week would be as effective as daily lessons, but it seemed easier for the school and regular classroom teacher if only daily schedules were considered.

It was decided that during twenty minutes first grade students could become sufficiently engrossed in a problem to produce some results and that twenty minutes absence from the regular classroom activities would not be an unreasonable demand of participants in the program. So the lessons were planned for twenty minutes duration.

A compromise between the minimum number of lessons the author felt would be necessary to produce some change in observable mathematical creativity and the length of time the author felt the experiment would be welcome in the school was used to determine the length of the program--fifteen lessons.

Consequently the author undertook some pilot studies to develop mathematical situations for a sequence of fifteen lessons, each of twenty minutes duration. These pilot studies are reported in the next chapter.

Chapter III

PILOT STUDIES

3.1 OUTLINE OF CHAPTER III

Two pilot studies were carried out primarily to develop mathematical situations for use in an experimental program of fifteen twenty-minute lessons for a group of six first grade students and also to gather other information pertaining to the experiment proposed to test the effects of participation in the experimental program. The primary purpose of the first study was to determine how to interpret in practice the requirement that the activities have a moderate amount of structure. The second study was undertaken to test specific activities having the right kind and amount of structure as determined in the first study, to ascertain whether one particular kind of general creativity program would be feasible with first grade students, to pilot the use of the Torrance Tests of Creative Thinking with first grade students, and to obtain videotapes for several uses. Both studies were done at the Prospect Street Elementary School, Lake Mills, Wisconsin.

Descriptions of Lake Mills and the Prospect Street School are given. Then each of the pilot studies is described and evaluated.

3.2 DESCRIPTION OF LAKE MILLS AND THE PROSPECT STREET SCHOOL

Lake Mills is a rural center of about 3,000 in south central Wisconsin. The school population is drawn from the town and neighboring rural community.

At the time of the pilot studies, grades K - 5 were located in the Prospect Street Elementary School. The principal, with the support of the district administration and staff, has been encouraging the teachers to experiment with student oriented and student directed classrooms. Many of the classrooms are of a less structured type, with the students, singly or in small groups, pursuing a variety of activities at the same time. Some classrooms are more traditional; the whole class does the same thing under the teacher's direction. In both studies, the students were drawn from classrooms of a less structured nature.

3.3 THE FIRST PILOT STUDY

The first pilot study took place between November 18 and December 11, 1968. The primary purpose of that study was to determine how to interpret in practice the requirement that the activities in the program have a moderate amount of structure. The other requirements set on the activities, and discussed in the previous chapter, were that they be open-ended geometric problems using concrete materials but not depending on a great precision of action, seeming to have no mathematical prerequisites other than counting and comparing integers or objects to 25, and generating observable behaviors during the process of solution.

The geometric content of the activities in this study included the examining polyhedra on simple properties such as number of edges or shape of faces, the finding an Euler path along the edges of polyhedra, and the coloring of planar maps.

Six first-grade students were chosen to participate in the pilot. Since one of them was frequently absent from school, the study actually involved five students. Three students were drawn from each of two classrooms. The classrooms were both essentially ungraded, each containing ten first, ten second, and ten third grade students. In both cases, the teacher was a mature woman with many years of teaching experience.

The basic materials used in this pilot study were colored plastic drinking straws and white chenille pipe cleaners. A pipe cleaner, folded double, can be inserted into two straws, forming them into a vertex. More than two straws can be made to meet at a vertex. This technique was suggested in a publication of the Ontario Institute for Studies in Education (1967, p. 35).

The first activity presented to the group of six students was making objects from the straws and pipe cleaners. Scissors were also provided so that the students could cut the straws to desired lengths. Once several objects had been made, and the students were familiar with the technique, an attempt was made by the author to focus attention on the similarities and differences among the objects. The plan was that this would lead to questions concerning the construction of objects having specified properties.

The students did make some interesting objects (cubes, pyramids, and more complicated unnamed three-dimensional figures, as well as some planar configurations), but the attempt at comparison produced few properties and little enthusiasm, perhaps because the students viewed the comparison activity as imposed by authority for no particular reason. It seemed that more structure in the activity itself was needed so that the students could focus on different properties as part of solving some problems. Therefore, three problem situations, having varying amounts of structure, were tried; two of them were couched in terms of a game.

At this point of the study, physical circumstances at the school made it necessary to use a very small room for the pilot activities. The size of that room meant that at most two students and the author could comfortable work in it at one time. The rest of the pilot study was conducted under these conditions, with students coming alone or in pairs to work on the problems.

The first problem was a two person game involving Euler paths on three dimensional and planar objects made from the straws and pipe cleaners. The first person to move tied one end of a string to a vertex of an object and laid the string along one straw, fastening the string to the vertex at the other end of that straw. That straw was now "used" in the sense that string could not be laid along it again. The second person took up the loose portion of the string and laid it along a straw extending from the second vertex and fastened the string to the other vertex met by the second straw. The two players alternated laying the string until one person could not move according to the rules. The person making the last legal move was the winner.

In order to encourage mathematical analysis of the object, the student was asked to build objects for playing the game and to decide who was to go first on each object in order to insure that he would win. This framework elicited some good means-ends analysis from the students and one creative response to some of the misconceived aspects of the framework. One student picked up a single straw and said, "I'm going to play on this and I'm going to go first." The student was reacting in an intelligent way to the undue emphasis put on winning. In retrospect, it would have been better to reward being able to predict who would make the last move on the basis of knowing who was to start and which object was to be used. Another possible restructuring could have been for one student to choose the object, the other player to decide who goes first.

The second problem was, "How many triangles can you make with only six straws and as many pipe cleaners as you need to fasten the straws together?" After some hints that to get more than two triangles, all straws must be used on the same figure, two very interesting restructurings of the problem were produced. Both were planar.

One response was the object pictured in (a) of Figure 3.1. It has two straws serving a purpose that could be done equally well by one.

The student did not, however, view his structure this way. He has four triangles (actually eight if one counts the larger ones) made from seven straws (actually six).

The other solution progressed as shown in (b) of Figure 3.1. The student made whole triangles and added them to his structure; he never added single straws. After reaching step 5, he was asked, "Do you think you could make something with just triangles that could stand up?" The response, step 6, was a delightful "failure," the addition of "legs." The student said he was fairly sure that his idea would not work, but tried it anyway, and laughed at his "failure."

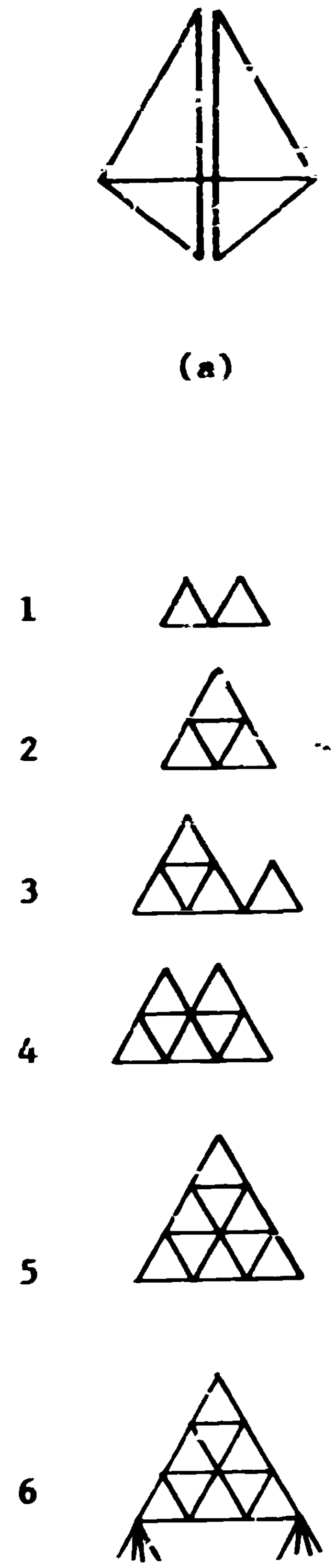

Figure 3.1

Two Responses to the Problem of Six Straws

The third problem was another two person game, this one based on the coloring properties of the plane. First, the two people took turns drawing lines on a piece of paper with the restriction that each line must have its ends touching either another line or the edge of the paper. When both people were satisfied with the drawing, each region into which the paper had been divided was marked by an "X". There seemed to be no difficulty with determining regions using the definition "a space you would have to cross a line to get out of." Then the first person colored a region with his color. Regions which touched in more than a corner could not be colored the same color. Under this rule, the two players alternated coloring regions, each with his own color, until one of them could not move. The person making the last legal move was the winner. There seemed to be no problem with the students' understanding the rules of the game, but the game was too hard for them to analyze. The strategy of simplifying the drawing, a parallel to the simplification of the straw and pipe cleaner objects which was done, did not occur to the students. Only one student liked this game: he liked to color under any circumstances. The other students seemed to dislike the structure put on their drawing and coloring.

From the reactions of the students to the situations piloted, it was decided that the right amount of structure was present in the string problem, but it was incorrectly framed as a game with too much emphasis put on winning. The triangle problem had the best framework, that of problem for its own sake, but may have been somewhat too structured since both of the interesting responses broke the requirement of only six straws. The other students were not sufficiently interested in the problem to do anything with it. The coloring game was too difficult and too structured for first grade students. The kinds of activities tested in the second pilot study were developed to have a framework similar to that of the triangle problem and the same amount of structure as the string problem.

3.4 THE SECOND PILOT STUDY

The second pilot study started on February 24, 1969 and ended on March 28, 1969. The purpose of the study was to test specific activities having the right kind and amount of structure as determined in the first study, to ascertain whether one particular kind of general creativity program would be feasible with first grade students, to pilot the use of the Torrance Tests of Creative Thinking with first grade students, and to obtain videotapes for several uses. These programs and tests were to be parts of an experiment to test the effectiveness of a program to increase the observable mathematical creativity of first grade students. As originally planned, pretests and posttests were to include problems of mathematical content and the Torrance Tests of Creative Thinking, Figural and Verbal Forms, in order to determine the effects of the program on the students' performance on each of the three possibly related areas of mathematical, figural, and verbal creative activity. As a further investigation into the possible relationship among creative activities, a program based on the Myers and Torrance Idea Books, Can You Imagine? and

For Those Who Wonder was to be given simultaneously with the mathematics program during the experiment, each to a separate group of six first-grade students, with the same battery of pretests and posttests given to each group (Myers and Torrence 1965a, Myers and Torrence 1966a).

During the second study, each of the programs was to be piloted with a group of six students. Students from two first grade classrooms were chosen for the study; six students from each classroom were taken as an intact group. Each of the classrooms was taught by a young woman.

The Torrance Tests

The Torrance Tests of Creative Thinking, Figural Form A, were given to three of the six students in the Idea Books instructional group (Torrance, 1966a). The tests were administered simultaneously and seemed within the range of interest and ability of the first grade students. The instructions were understood and followed, and sufficient work was done during the time allowed to make grading possible. The three test activities all required the student to complete drawing a picture in which some lines had been drawn. Only the second two of the three activities of the test were used in order to accommodate to the shorter interest span of the younger child. Activity one was eliminated because it was the only one which could not be scored on the fluency dimension.

The Torrance Tests of Creative Thinking, Verbal Form A, were administered individually and orally to the same three students (Torrance, 1966c). Again, accommodating the younger child, only two activities were used: Product Improvement and Just Suppose. These were chosen on the basis of the recommendations of Torrance for just such an accommodation (Torrance, 1966e, p. 10). These tests were not well received. The students were not hesitant to speak during the test, but seemed unable or unwilling to talk about the hypothetical situations posed in the test and described by this author. Thus the question, "What do you think might happen differently if every cloud had a string hanging from it which came all the way to the ground?", posed while showing the student a picture of such clouds, elicited few responses, none more imaginative than "I would climb the rope," and some irrelevant ones, such as "rain" and "snow". It seemed to the author that although the first grade student can at times be highly imaginative verbally, he has great difficulty distinguishing between the real and the imaginary, and consequently cannot easily give relevant responses to hypothetical situations. So few responses were given to the verbal tests that one could not expect to derive meaningful scores from them.

On the basis of the reception given the two forms of the Torrance Tests of Creative Thinking, it was decided to use only the Figural Form during the experiment.

Idea Books Program

The Idea Books instructional program produced responses similar to those elicited by the Torrance Tests of Creative Thinking, Verbal Form, with which the Books share much format and content. This reaction was unfortunate in terms of the experiment as first envisioned, but was somewhat anticipated in the light of the general comments in the Teachers' Manuals for the Idea Books. There it is suggested that the lessons from these Books be used when classroom circumstances have provided adequate warm-up and lead-in activities (Myers and Torrance, 1966b, pp. v-vi). The attempt to use the lessons at a fixed time of day for a fixed length of time under the direction of an adult who was not the regular teacher and in a setting other than the regular classroom was not successful. The unsuccessful nature of this attempt was so soon evident in the pilot study that the Idea Books instructional program was piloted only two days and then it was decided to eliminate the general creativity program (Idea Books) from the experiment.

Mathematical Creativity Program

Six different activities, each encompassing several lessons, were piloted with a group of six students. They are discussed in the order in which they were presented to the students. The geometric content of the activities in this study included the exami ıtion of rods of different lengths for the purpose of making combinations of rods having equal length and representing the different lengths using pictures or numbers; the examination of planar drawings and tesselations for paths with properties such as longest or shortest length and existence of self-intersections; and the making of polyhedra and surfaces and the examination of these objects on simple properties such as number of edges or number and shape of faces. The source of the ideas leading to some of the activities based on tilings of the plane is a publication of the Association of Teachers of Mathematics (1968, pp. 104-110 and 131-142).

The first activity was based on Cuisenaire Rods, a manipulative device which was not familiar to the students. These rods have a square 1 cm. x 1 cm. as cross-section and measure one of ten lengths; 1 cm., 2 cm., ..., 10 cm. Two rods are the same length if and only if they are the same color. The rods were referred to by their colors, not by numbers, during the entire activity.

The students were each given a small burlap sack containing several of each of the ten rods and asked to guess what was inside by feeling but not opening the sack. They were encouraged to guess how many different sizes the objects were. Then they were allowed to open the sacks and to determine whether they guessed correctly the number of different sizes. As expected, the students grouped the rods by color and counted the number of groups. The rest of that twenty-minute period and the whole next period were devoted to making structures of the students' choice from the rods.

Several interesting structures were made. The students tended to stand the rods on end, making "walls." One girl made "steps," using one of each length rod, all standing on end. She then proceeded to level the "steps" in order starting from the vertical 9-rod by adding a rod to each step so that the column became equal to longest rod. She told the author that she knew which rod to add to the next step by looking at the "backwards" order of the rods in the steps.

The third day, the students were asked to try to draw pictures of the structures they built so that someone could build the same structure using only the picture as a guide. It was expected that someone would have tried tracing the rods as a means of representing them, but no one did. The drawings ranged from some fairly representational ones, which showed juxtapositions and relative size well, to some in which only the idea of the object was conveyed, for example a boat made from the rods was pictured as a boat with slanting sides and sails.

The fourth day, the students were asked if they thought that instead of using colors to name the rods, they could use numbers. It was expected that the numbers one through ten would be suggested, although possibly not in the order which makes it possible to demonstrate addition and subtraction by laying the rods end to end. The first number suggested for a name was "283!" The author picked up the shortest rod and asked "If I call this color number "1," what color would I call number "2?" A few such leading questions produced correct responses, but the forcing of the situation in that way was terminated very quickly because it was obvious that the students had not had enough experience with the rods to be able to respond with understanding.

The first activity was considered successful, even though the third and fourth days spend on it were less than resounding successes, because the first two days had produced good responses and because the author felt that given more time to ponder, the students could have done interesting things with the problems of the latter two days.

The second and third activities involved paper printed with one of two patterns shown in Figure 3.3.

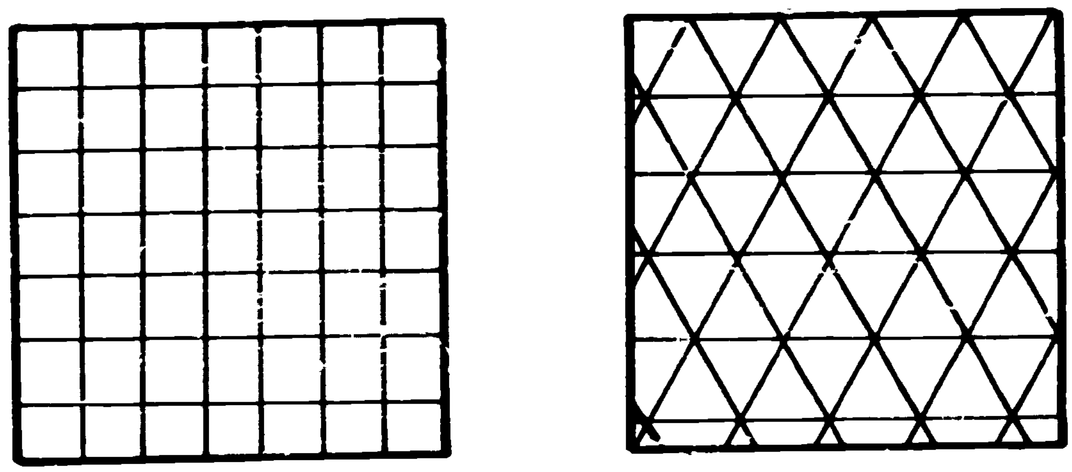

Figure 3.2

Patterned Papers used in the Second Pilot Study

The second activity was a group rather than an individual problem. A path leading from some inside vertex to the edge of the pattern was drawn along the lines of one copy of the pattern (the first example was done by the author). A clear plastic sheet had the same path drawn on it so that if the sheet were properly placed over the pattern, the paths would coincide. One student was not allowed to see the path, but was given a paper with the pattern and a pencil. The other students' job was to convey through words alone enough information to the isolated student so that he could reproduce on his paper a path which would coincide with the one on the plastic sheet.

Howard and Templeton report that left-right discrimination is not well developed among six year olds; they scored just above chance in response to the items in one experiment (1966, p. 290). On this basis it was expected that this second activity would be very difficult for first grade students, and it was. Surprisingly, some of the students, by the second day of this activity, were consistently giving information in terms of length and direction of line segments. Other students, however, had great difficulty reproducing a path when it was in front of them and when a plastic coincidence sheet also was available.

The third activity, again using the patterned papers, was based on the various properties which a route connecting two vertices and drawn on the pattern lines could have. Each student chose two vertices on his piece of patterned paper and was asked to draw a line connecting them which was special in some way. All the students drew "shortest" ways; some even noticed that the two paths on the squared paper which together outline a rectangle were the "same" in some way. But no one, much to the surprise of the author, named such properties as most or fewest number of turns. More sophisticated properties such as with or without self crossings were introduced by the author, but with the exception of one student, no one responded to the challenge to "cover the whole pattern while going from this point to that point and not retracing any part of your path." The student who did respond gave the problem a good try, but quit after seeing how hard it was to really accomplish that end.

This third activity seemed to involve a level of mathematical sophistication and abstractness not appreciated by the students, as indicated by their general lack of interest in all but the simplest properties of the paths.

Trying again for Euler lines in a planar setting, the author suggested the fourth activity to the students. Each student was to draw a picture, of any kind, on blank paper with the only restriction being that he could only draw one line; that is, once his pencil left the paper, he could not put more marks on the paper. Then each student was to exchange papers with another student to see if he could draw with only one pencil stroke the same picture his partner had.

The author anticipated some difficulty in the second half of this activity, the drawing under the restriction of a figure made by another, but the actual outcome was worse than expected. The students either did not follow the restriction in making their first drawing or else in following it, made such a scribbled mess that it would be impossible for anyone to reproduce it. This activity was abandoned during the first 20-minute period in which it was tried. As with the coloring game in the first pilot study, drawing on blank paper seemed to be interpretted by the first grade students as subject to no restrictions.

As materials for the fifth activity, equilateral triangles and squares, $1\frac{1}{2}$ inches per side, from white card stock were provided. The students were presented with these, tape, and several closed surfaces made by the author, including the surfaces bounding four of the Platonic solids, and were told they could make anything they liked taping together the shapes.

The responses were exactly as anticipated: some students made planar forms (flowers, etc.); some students made objects of their own choosing (houses, etc.), and some students attempted to duplicate one of the surfaces made by the author. When a student was not involved in duplicating structures presented by the author, he me very interested in making objects which were in some way di t from those made by the other students.

On the third day of this activity, the author trie ave each student request a specific number of squares and triangles with which to ake his object. This was supposed to lead to a consideration of the variety of constructions possible with a fixed number of each shape. As with the triangle problem in the first pilot study, this fixing of construction materials was too stringent a requirement for most of the students. It was, therefore, not pursued.

Except for the unsuccessful attempt to add some structure of a numberical nature to the proceedings, the fifth activity was very successful in the expected fashion. The mild pride and competitiveness which motivated the students to try to make objects different from those of their peers was not anticipated, but was a good turn of events, since otherwise the author, according to plan, would have had to try to encourage such differences. According to the principles of the previous chapters, for example preference for intrinsic motivation, the actual source of this striving to be different was better than the planned motivational source.

The sixth activity piloted during the second study was somewhat the same as the first activity of the first pilot study; namely, making structures from straws and pipe cleaners. Several changes in the presentation made the activity more successful the second time. First, because this activity was not the one which introduced the students to the author, but followed several reasonably structured activities, it seemed that the students had a better sense of what kind of responses the author was seeking. Second, the author, at the time of introduction of the materials, showed the students several objects made with straws and pipe cleaners (the octahedron and icosahedron were included). This was similar to the introduction to the fifth activity of the second pilot study and was not done in the parallel activity in the first study. The third difference was that the students were not provided with scissors for the purpose of cutting the straws to desired lengths. This omission reduced the flexibility of the materials and the kinds of activities open to the students, thereby adding a bit more structure to the possible outcomes of the activity.

The responses to this modification of the original activity were highly similar to the responses to the fifth activity: Some students made only planar objects, some made objects of their own choice, and some tried to copy the structures presented to them. Again, the students tried to make objects different from those made by other students. The same evaluation which applied to the fifth activity can be applied to the sixth: It was a success.

One student made a comment during the sixth activity which could not be pursued then, because of the pressures of time, but which became a focal point of some of the activities of the experimental program. The student, pointing to the octahedron made of straws and pipe cleaners, said "That's the same shape we had before." He was referring to the octahedron made of the white card triangles. That a first grade student should see the similarity of the three-dimensional shape of two objects, one closed and one open, and when one of which was not even present, was not anticipated by the author, but was most welcomed.

Videotapes

Between the first and second activities piloted for the mathematical creativity program, on March 4, 1969, a one-day activity was presented individually to the six students in the mathematical creativity instructional group, two students from the Idea Books instructional group, and two other members of the same classroom as the first six students. Each student worked alone on a problem in the presence of the author and the entire proceedings were videotaped. The problem was "How many times do you think you could trace this triangle (equilaterial, $1\frac{1}{2}$ inches per side) on this paper (blank, $4\frac{1}{4}$ x $7\frac{1}{2}$ inches) so that you could cut the triangles out?" Because this mathematical activity and videotaping were a piloting of part of the test instrument, a discussion of this activity and an explanation of the purpose of the videotapes are in Chapter V.

The experiment as originally planned was modified to account for what was learned during the second pilot study. The Idea Books instructional program was eliminated as a treatment in the experiment.

Because the Torrance Tests of Creative Thinking, Verbal Form, did not produce results worth measuring, only the Torrance Tests of Creative Thinking, Figural Form, were used in the experiment as a measure of general creativity.

Of the activities piloted for the mathematical creativity program, the evaluations of their success can be briefly summarized as follows: The second activity was too difficult; the third activity was not interesting enough to the students; the fourth activity was too structured; and the first, fifth, and sixth activities were in some respects very close to perfect. From these successful activities, were formed the activities and daily plans used during the experimental program. The method and rationale of the formation and an account of the actual program are in the next chapter, Chapter IV.

Chapter IV

THE EXPERIMENTAL PROGRAM

4.1 OUTLINE OF CHAPTER IV

An experimental program consisting of fifteen daily lessons each of twenty minutes duration was constructed from the activities tested during the first and second pilot studies. In the first main section of this chapter, the criteria by which these activities were selected are given and each of the activities originally chosen is described; the original sequencing of the activities is also presented. The experimental program was given starting April 16, 1969, and ending May 6, 1969, to a group of six first grade students from the Poynette Elementary School, Poynette, Wisconsin. A description of Poynette and the Elementary School is in the next section of this chapter. The final section is an account of the actual program, noting and explaining the changes made in the activities and sequencing.

4.2 THE ACTIVITIES CHOSEN

In Chapter II, the mathematical situations to be used in the program were described as open-ended geometric problems having a moderate amount of structure, using concrete materials but not depending on a great precision of action, seeming to have no mathematical prerequisites other than counting and comparing integers or objects to 25, and generating observable behaviors during the process of solution. All activities piloted satisfied these requirements with the possible exception of the moderate amount of structure. Some additional characteristics were required of the problem activities chosen for the experimental program.

One of the additional requirements was that each activity had to be an individual rather than a group problem. One group problem dealing with communicating information about paths on squared or triangulated paper had been tried with some success during the second pilot study and on a few other occasions the author had tried to encourage some minor forms of group or partner activities. One the whole, these efforts did not elicit behaviors significantly more mathematically creative than did the individual problems, and they did involve coping with group behaviors. Because the primary interest of the author was in individual creative mathematical activity and because group activities seemed more difficult to direct and seemed to give no benefit of additional creative responses, the decision was made to have only individual problems in the experimental program.

A second condition imposed on the activities was a restriction of content. The author felt that the more restricted the total content of the program was, the greater the chances would be for the student to see interconnections and to have illuminations. Many of the activities piloted did share much content; they involved incidence-type relationships pertaining to arrangements of triangles and quadrilaterals, especially equilateral triangles and squares. Except for the coloring game of the first pilot study and the Cuisenaire rods activity of the second pilot study, all of the activities involved this content either directly, as in the activities using either the patterned paper or the equilateral triangle and square cards, or somewhat less directly, as in the activities based on the straw and pipe cleaner constructions. Although eliminating the Cuisenaire rods activity meant the loss of a successful activity from the pool of possible activities, it was felt that the resulting smaller content area compensated for the loss. Consequently, it was decided that all the problem activities should involve the incidence-type relationships pertaining to arrangements of triangles and quadrilaterals, with special emphasis on the equilateral triangle, the square, and one additional shape, the rhombus formed by two equilateral triangles. Throughout the remainder of this chapter, the kind of rhombus formed by two equilateral triangles will be called a diamond.

Eight activities were planned for the experimental program. They are described in the following paragraphs in the same order as they were expected to occur in the program. For each, the materials, the problem components, and the expected number of lessons are given.

The first activity was to be a tiling activity. The materials were a collection of painted cardboard tiles, approximately 100 each of red equlateral triangles, blue squares, and green diamonds, each shape 1 1/2 inches per side; unlined white paper 5 1/2 x 8 1/2 inches; and ball point pens. This activity was to start with finding ways to completely cover a sheet of paper with the tiles so that the tiles do not overlap. Then the set of all possible coverings was to be classified according to the shapes used in each and this information recorded on a chart. Using the chart as a guide, the students were to try to make examples of each specific type of covering; in order to determine whether each type of covering had been made, a means of recording the patterns of the coverings was to be devised. It was expected that this activity would take three days.

The materials for the second activity were several hundred each of equilateral triangles, squares, and diamonds, each shape 1 1/2 inches per side, cut from white card stock; and tape. The problem was to make surfaces by taping together the various shapes edge to edge. Some closed surfaces were to be presented to the students as examples, but no restrictions on dimension or closure of the surface were to be made. Two days were to be given to this activity.

The third activity was to use the same materials as the second. The problems were to discover the number of different configurations possible using a fixed number of triangles, squares, or diamonds placed edge to edge and to help the teacher make charts recording the discoveries. Two days were allotted to the third activity.

The fourth activity also was based on the same materials as the second activity with the addition of paper measuring 5 1/2 x 8 1/2 inches. The problems were first, to make new shapes from the equilateral triangles, squares, and diamonds, each shape having the property that it could tessellate (that is, that many copies of the shape could cover a sheet of paper); and then, to help the teacher make a chart with examples of all the new shapes having this property. Another two days were allotted to the fourth activity.

The materials for the fifth activity were plastic drinking straws of three lengths: 10 1/4, 4 1/2, and 5 3/4 inches, and white chenile pipe cleaners. One day was to be devoted to learning how to make structures from these materials. Some examples were presented to the students, but no restrictions on the structures were to be made.

The sixth activity, to which two days were to be devoted, was to be based on the structures and materials from the fifth activity, with the addition of string. The problem was to build structures which have Euler lines.

The seventh activity was to use the same materials as the fifth activity. Problems of the nature of "How many triangles can you make with six straws?" were to be given. Only one day was allowed for this activity.

The eighth and final activity was a combining of the second and fifth activities. The problem was to make the same shape object, of the student's choice, from the cards and tape and from the straws and pipe cleaners. Two days were left for this activity.

The initial sequencing of the activities had the property that some of the later activities shared either materials or problem components with activities which were earlier but not immediately preceding them. For example, activities one and four involve tiling, activity eight combines activities two and five. This aspect of the sequencing was intended to aid the unconscious processes of incubation, intimation, and illumination and to encourage transfer.

A description of the school in which the experimental program was given is in the next section of this chapter. An account of the actual instructional program follows. The journal of the program activities, more detailed than the account in this chapter, is in Appendix A.

4.3 DESCRIPTION OF POYNETTE AND THE ELEMENTARY SCHOOL

Poynette is a small town of approximately 1,100 people in south central Wisconsin. The school population of 245 in grades K-5 is drawn from the town and neighboring rural areas. Most of the families in the school district are lower middle class or middle class. Many of them are farmers or are employed by small businesses which serve the farms or by food-processing companies. There is some exchange with the state capital, Madison, which is thirty miles south; some of the residents of the area commute to Madison and some of the teachers in the school commute from Madison.

At the time of the experiment, during the construction of a new middle school, many of the early primary grades were housed away from the elementary school, in church basements and fire houses, and the fourth through seventh grades had rooms in the Elementary School. Two first grade classes shared an older self-contained frame building adjacent to the Elementary School. From one of these classes were drawn the students to participate in the experimental program. The method by which the students were selected is described in Chapter V, THE EXPERIMENT.

The classroom from which the students were selected was essentially self-contained. The teacher was a mature woman with many years of teaching experience. The classroom activities were primarily teacher directed and seemed somewhat formal.

4.4 THE ACCOUNT OF THE EXPERIMENTAL PROGRAM

The experimental program was given daily starting April 16, 1969, and ending May 6, 1969. Wisconsin State Law prohibited the author from doing the actual teaching because the author is not a certified elementary school teacher. The teaching was done by Mrs. Carolyn Gornowicz, a certified teacher employed by the Wisconsin Research and Development Center for Cognitive Learning.

Mrs. Gornowicz prepared for her teaching of the program by reading preliminary versions of Chapter I and Chapter II of this thesis and by discussing the activities planned for the program with the author. She also became acquainted with the criteria describing aspects of creative mathematical activity which are part of the test instrument reported in Chapter V of this thesis.

The day after Mrs. Gornowicz saw the card triangles which could be taped together to form surfaces and some of the surfaces bounding the Platonic solids, the author noticed models of the tetrahedron, octahedron, and icosahedron on Mrs. Gornowicz' desk. She had made them at home because she was curious about their construction. The author had been satisfied by talking to Mrs. Gornowicz that she was the kind of teacher who naturally acted in the ways specified in Chapter II of this thesis. Her questions about the activities had indicated that she was intrigued by the kind of mathematics to be done in the program. The construction of the models was a surprising and pleasant incident which again indicated that Mrs. Gornowicz was the right kind of teacher for the program.

During the lessons, Mrs. Gornowicz was in charge and the author acted as her aide. Each lesson began with a brief statement or restatement of the problem for that day. Mrs. Gornowicz encouraged the students to participate in this part of the lesson by asking them questions which would stimulate the students to observe properties of objects or examine conditions of the problem. This student participation was also expected to act as a source of motivation.

As the students worked, Mrs. Gornowicz and the author moved around the room observing and encouraging the students. Sometimes a student would seem to need some personal attention before he would select a particular problem. Often a student would become frustrated if his first approach was not working; much time was spent by Mrs. Gornowicz and the author encouraging students to regard their work as tentative, not final, and to change aspects of their work in order to achieve the desired end. This encouragement ranged from close personal attention and help in construction to the offering of statements like, "Sometimes you have to take a thing apart half way in order to make it be what you want." This was the principal preparation or manipulation technique emphasized in the program.

The verification of the students' results usually toook the form of ascertaining that two objects corresponded in pattern or shape. Most of the time, verification was done by the students as part of the process of constructing the objects. In those few cases of a student being satisfied with an object which did not display the required properties, Mrs. Gornowicz or the author would try to encourage the student to verify his results. This was done by asking the student what the requirements of the problem were, and then examining the object in question. The adult would always begin by indicating ways in which the object satisfied the requirements. Often, if the student indicated that he realized that the object did not satisfy a particular requirement, the adult left the verification process to the student at that point, and did not require the student to change his work. This was done so that verification with the teacher participating would not be considered as a form of punishment by the student.

In the evaluation of the daily activities and preparation for the next day's lesson, the relationship between Mrs. Gornowicz and the author became more of a partnership with Mrs. Gornowicz contributing an understanding of the students and a feeling for how things were going or would go and the author contributing a knowledge of the mathematical relevance of the various activities, especially the way in which these activities were related to each other. All decisions made during the course of the program to modify, eliminate or resequence the activities were made by the teacher and the author as a team.

A brief account of the program follows; the journal of the program, giving daily plans, events and evaluations, is in Appendix A.

The first five lessons of the program were spent on the first of the planned activities, which was primarily tiling using equilateral triangles, squares, and diamonds. This activity, from as early as the

the first lesson, took longer than the planned three days. It was considered better to satisfactorily complete fewer activities in the program than to rush the students. Consequently, two activities were eliminated from the original list--the third and the seventh--both activities concerned with the constructions possible with a fixed set of materials This type of activity had been only occasionally successful in the pilot studies and was therefore the first kind of activity to be eliminated. Because so much time had been spent tiling, it was decided that the second activity should present as great a change of materials as possible within the range of planned activities. The sequencing decided upon for the remainder of the program was making straw and pipe figures and finding Euler lines (combining two activities), making surfaces from card stock shapes, making the same shape from both kinds of materials, finding new shapes which tessellate. This sequencing, which shared with the original sequencing the property that the student might transfer elements from earlier activities to the later ones, was the actual one followed in the program.

During the first lesson, the students tried to cover sheets of paper with the tiles. The second lesson began with the making of a chart which classified possible coverings into seven types depending on the shapes used, then the students returned to their tiling efforts. In the third lesson the idea of making drawings of the coverings in order to preserve what they looked like was introduced; the third, fourth and fifth days were spent making coverings and drawing pictures of them. The pictures were hung on a chart, a representation of which is given in Figure 4.1.

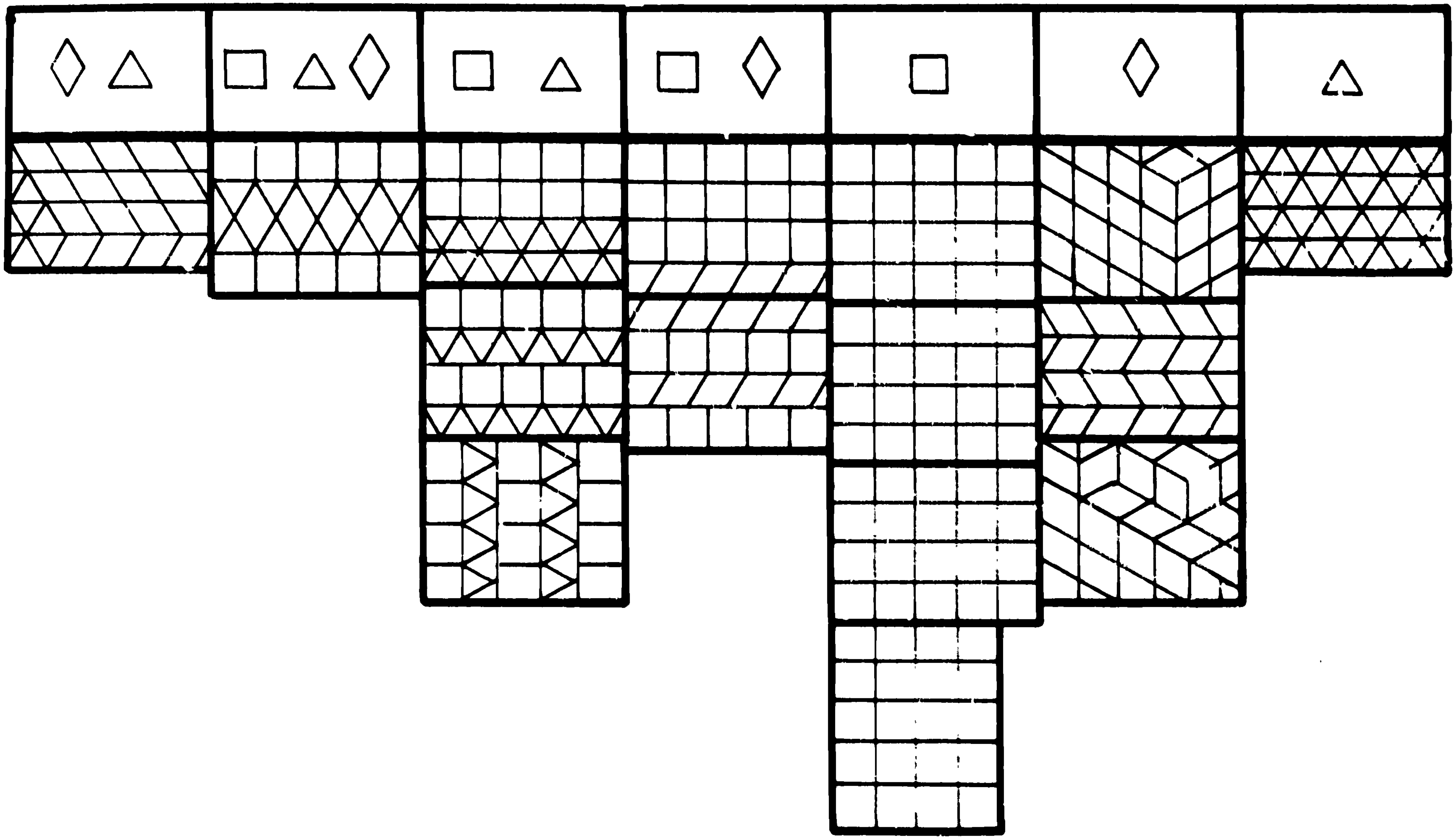

Figure 4.1

Summary Chart Made During Activity One

At its very best, a creativity-encouraging situation should provide stimulation to the teacher as well as the students. Occasionally this happened in the program, as for example this incident on the fourth day. Two pictures were hanging under the category triangles and squares; both had four horizontal rows, two of squares and two of triangles. One had both the rows of squares as top rows, the other had the rows alternating. A student who had made one of them was asked if both pictures were the same. The reply was that one had triangles "there" (pointing to the second row), and the other had squares.

This answer led the teacher and the author to think about the problem the students were having in making a covering with all three shapes. One basic factor in this problem was that unless the squares were used in complete rows, a problem of angles presented itself. Despite encouragement, most of the students found it very difficult to remove tiles in order to complete a cover; and since the attempts to make coverings with all three shapes invariably led to this problem, it seemed that without some hints, no such covering would be made. There was a hesitancy to offer hints, however, because neither the teacher nor the author were sure that any of the possible hints which they had thought of would be a natural extension of what the students were consciously doing. Then the comparison made by the student indicated some awareness of rows. Upon examination of the coverings already drawn, the teacher and the author found that all three shapes had been used in rows. At the beginning of the fifth lesson, the coverings were examined by the students for the existence of rows and then the teacher suggested that perhaps putting the shapes in rows would help in making a covering with all three shapes. One student took the hint and did make such a covering.

As she was working, the student commented that it was simple to make a covering with all three shapes. It was pointed out to her that all things are simple if you know how to do them, but that if you say "It's simple," to someone when you know how but he does not, then you may hurt his feelings. Not much understanding was expected of this rather abstract and hypothetical verbal comment, but within five minutes, the student was heard explaining the principle to the others. Very soon, "simple" and its close relative "simple dimple" became "magic words" which the students would not say, but would make a big fuss over not saying. In every lesson after this fifth one, some student recalled the fact that they were not supposed to use the word "simple." It was surprising to find how quickly and with what seemingly real comprehension the students adopted a principle communicated verbally to them.

Returning to the activities of the fourth day, the student who made the covering which had alternating horizontal rows of squares and triangles commented that she was making "houses." The comment was heard by another student, who looked at her work and then produced a covering having alternating vertical rows of squares and triangles.

The second activity was to begin with making straw and pipe cleaner constructions and to progress on the second, or at the latest the third, day to constructing figures which have Euler lines. An octahedron and tetrahedron were presented as examples of possible constructions, but it was made clear that the students did not have to try to copy them. It turned out that on both the second and third day of this activity, some very worthwhile efforts were being made at copying the octahedron, so rather than disturb these efforts, the Euler line problem was never introduced.

One of these efforts stretched over two days. A student began her copying of the octahedron by counting the number of triangles it had. She then proceeded to make eight separate triangles from the straws and pipe cleaners. At the end of the first day, when she had five of them done, she was heard to remark, "Now how am I going to put these together to get that?" This problem arose during the second day, when having finished all eight triangles, she started to put them together. She became disturbed at the fact that her construction had two straws where the model had only one. In order to exactly duplicate the octahedron she needed to either dismantle some of her triangles or abandon them. Both methods required her to backtrack a bit, and this seemed very hard for any of the students to do (as in the tiling problem). The teacher patiently helped and encouraged the student, and just as the lessons ended, she finished; it seemed she would burst with pride.

A second student tried to copy the octahedron using a different technique. He made one triangle, placed it one the octahedron, added a straw, again placed his object on the octahedron, added another straw and continued in that fashion.

The third activity: making surfaces from card stock equilateral triangles, squares, and diamonds, was begun on the ninth day of the program. Several examples, including a cube, octahedron, and tetrahedron, were shown but, as in the previous activity, it was stressed that the students could make any sort of object from the materials. Two days were spent on this activity.

As one student worked, she remarked that she was doing the same thing with these materials as had been done with the tiles; she said that she was trying to tape them together so that they were flat and no table showed through. The activities had been sequenced and the mathematical content had been narrowed to increase the chances for exactly this kind of spontaneous recalling of an earlier activity.

Another student introduced a new structure to the group--a crown, made by taping together a row of squares, joining the first and the last square, and then taping a triangle to the upper edge of each square. Several other students picked up on this idea and made crowns or modified the construction and made bracelets.

During this third activity one student said, "This is play." When asked what was work, the student replied that the tiling activity was work. The teacher asked for the difference between the two, and was told that, "Work is hard and play is fun." This definition was surprising not only because it was unexpected, but because of its succinctness and its agreement with the views of some adults, especially some social scientists. It was interesting expecially because in the discussion which followed among the students, it became clear that although work and play may be opposites, "hard" and "fun" were not necessarily opposites, but seemed to be the primary aspects of the activities in question.

The fourth activity, making two objects with the same shape using the card stock shapes for one and the straws and pipe cleaners for the other, was introduced by the teacher through a comparison of two tetrahedra, one of each kind of materials. When the teacher asked for ways in which the objects were different, a number of properties were offered by the students: size, colors, means used to hold the object together, materials and openness ("can stick your hand through"). When the teacher called for similarities, the existence of triangles was mentioned; the teacher introduced the word "shape." The number of properties used spontaneously by the students to compare the tetrahedra was much larger than the number used by them to compare two coverings on the first day of the program. It is not clear whether this increase is due primarily to the nature of the objects being compared--the teacher and the author feel that perhaps the tetrahedra had more differences of a perceptible nature--or to participation in the experimental program and familiarity with the materials. Whatever the cause, some of the properties offered were not expected, for example means used to hold the object together and openness.

The two days spent on the fourth activity were followed by two days devoted to the fifth activity, making new shapes which could tessellate a planar section using the three card stock shapes. This activity was the most difficult one in the program, and required a great deal of explanation. A combination of luck and some examples of new shapes presented by the teacher resulted in the finding of several new shapes. These were put on the chart shown in Figure 4.2.

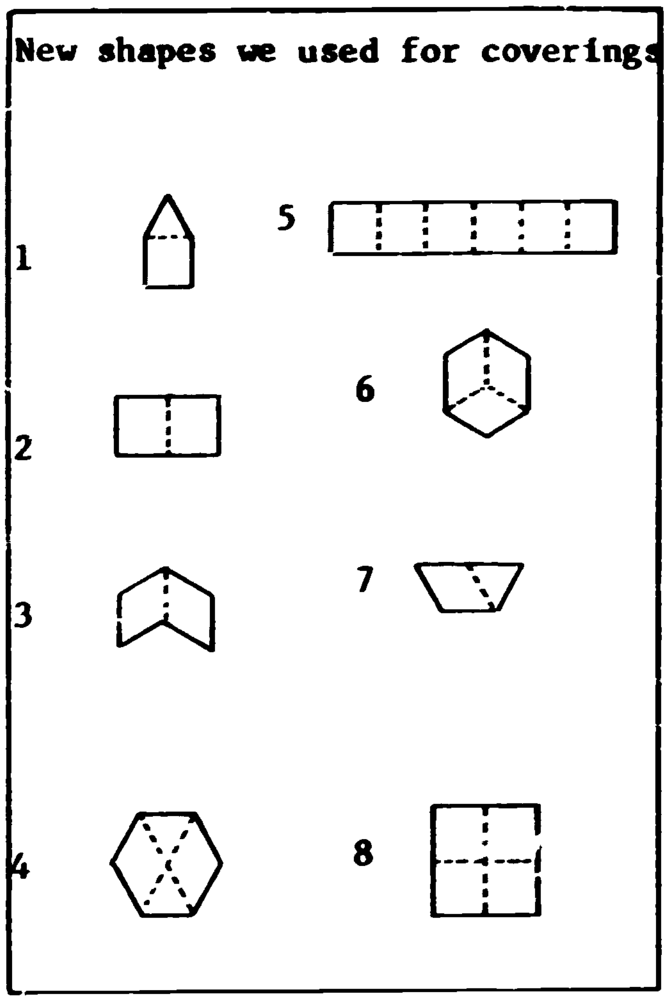

Figure 4.2

Summary Chart Made During Activity Five

Although it may appear to the unknowing observer that two of the new shapes are the same, the student who made the second hexagon was quite certain that they were different because one was made from two diamonds and two triangles and the other from three diamonds. A difference which is somewhat mathematical or not mathematical at all was often sought by the students as a means of distinguishing their work from that of the other students. The desire to produce something different arose naturally during the program and often produced amusing responses, as in the labeling of many identical structures with different names such as "house," "barn," "church," and "dog house."

The fourth and especially the fifth activities seemed quite difficult for the students. It was considered detrimental to the aims of the program to force prolonged attention on these problems, so consequently each was pursued only two days and the last of the fifteen lessons was made a free period with both the card stock shapes and the straws and pipe cleaners available.

This fifteen lesson program was given as the treatment in an experiment to test whether this treatment would increase the observable mathematical creativity of the participants. The measures of observable mathematical creativity are described in the next chapter.